Revista Brasiliense de Engenharia e Física Aplicada

Vol. 4, No. 1 - Março 2019

Astronomia:
Lentes gravitacionais

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ISSN 2526-4192

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Guanabara Bay

Proposals for a Territory of Exclusion Born from Paradise — Part I, The Present-Day Mess

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Received: _03/05/19_ / Accepted: _04/02/19_ / Published: _04/30/19_.

Abstract: Exclusion Territories are geographical areas under the action of degenerative environmental phenomena of anthropogenic origin, which compromise quality of life in general. One of the greatest examples of such areas is the Guanabara Bay and its surroundings, the scene of some of the worst disastrous incidents and locale of frequent episodes of human misery. This article presents a brief description of the main characteristics of the region, providing some technological suggestions of biogeographic recovery to be adopted by public policies that intend to align themselves with the good practices of ecological economy, sustainability and quality of life. The work falls within the context of macro-engineering *cum* eco-innovation applied to the preservation and management of water sources and water bodies that serve productive purposes as natural niches and breeding grounds.

Key words: Exclusion Territories, Guanabara Bay, waste management, quality of life.

Resumo: Territórios de Exclusão são áreas geográficas sob ação de fenômenos ambientais degenerativos de origem antropogênica, os quais comprometem a qualidade de vida em geral. Um dos maiores exemplos de zonas desse tipo é a Baía de Guanabara e seu entorno, palco de alguns dos piores incidentes desastrosos e de frequentes episódios da miséria humana. O presente artigo descreve sumariamente as principais características da região, fornecendo algumas sugestões tecnológicas de recuperação biogeográfica a serem adotadas por políticas públicas que pretendam alinhar-se às boas práticas de economia ecológica, sustentabilidade e qualidade de vida. O trabalho se insere no contexto da macroengenharia *cum* eco-inovação aplicada à preservação e à gestão das fontes hídricas e dos corpos de água que servem a propósitos produtivos como nichos naturais e criadouros.

Palavras-chave: Territórios de Exclusão, Baía de Guanabara, gestão de resíduos, qualidade de vida.

1. Introduction

The 1979 James Bond epic ***Moonraker*** featured the awe-inspiring scenery of beautiful Rio de Janeiro as well a nearby secret locale X where aero-spacecraft transports are launched. Gathered genetically perfect youths (representative of all humankind) were to ride these craft and eventually dock with an Earth-orbiting space-station. There, they would await the impending deliberate extermination of all the unfit and ugly human beings isolated below before their triumphant return to Earth's biosphere for its repopulation! Brazil's ever-failing leadership might wish they could be isolated from those, both rich and poor, dwelling next to a polluted and offensive bay!

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This work was sponsored by
UNIDESC Centro Universitário

The Guanabara Bay (in *Tupi* Indian language, Guanabara means “the breast of the sea”), the old postcard slogan of one of the most beautiful cities in the world, is today an aquatic chemical and rubbish dump, a great marine cesspool in the open. It is nowadays a symbol of social exclusion in Brazil, with 50 of its 55 tributaries becoming black ditches draining 15,000 liters of untreated sewage per second. Virtually all tributaries suffered corrections of flow, which accelerated the process of natural sedimentation. Among the more than ten million inhabitants living in its surroundings, it is estimated that at least one third live in slums (shantytowns), and another third live without basic sanitation and minimal urban infrastructure. This population makes Guanabara Bay (GB) one of the most important coastal environments in the country from a social, economic and environmental point of view [1-3]. Despite its beauty, importance as a tourist and economic center (Grande Rio megalopolis), GB sometimes stinks and often is an eyesore. Without a dry season, ~2.1% of Rio de Janeiro State — including the state capital and Niteroi — are blessed with Koppen’s Af climate [4]. Like most modern-day cities Rio de Janeiro is domed by an aerial heat island [5].

GB was also named *Rio de Janeiro* (“*rio*” is the Portuguese word for “river”) by André Gonçalves, Captain of the Portuguese fleet arrived at the waters of the *Tupis*. Obviously, it is foolish to think, as many people want, that Portuguese navigators would have confused GB with a river, since the word "*rio*" was used at the time to designate any mass of water, and that Portuguese navigators, highly competent and perhaps the greatest in history, were extremely experienced cartographers, cosmographers and observers, who would scarcely make such a mistake.

Guanabara Bay bathymetry resembles a sea-shell. It was filled with seawater first approximately 6,000 years ago as the post-Ice Age ocean rose [6]. The geographical beauty of the region has long been exalted. As Saint-Hilaire (1779-1853) said,

"*Qui serait capable de décrire les beautés de la baie de Rio de Janeiro, ce port qui, de l'avis de l'un de nos amiraux les plus savants, pourrait contenir tous les navires de l'Europe*?"

Why has an ecosystem of such importance and obvious charms been left aside, absolutely abandoned by the authoritative public power? There is a perverse combination of factors. On one hand, the low-level of education of the people precisely allows people with limited perhaps shriveled humanistic capacity to be raised to power, in addition to the constant presence of unbridled corruption. After years of anthropological and social studies, we are led to theorize that poverty imposed for a long time becomes a state of mind that completely dominates a population; malleable people adapt to misery in order merely to survive, as if they were made insensible by political morphine, by empty promises, and do-nothing concretely to change; they get used to the worst health and sanitation scenarios, and re-elect the most cynical politicians for crumbs called aid, but which are actually payoff-like handouts. We suppose this is how Latin governments impose their sovereignty: establishing poverty as a state of mind, while dominators do everything to keep things as they are or escape to far-pleasanter places, someday including a nearby Space Station or even a terraformed Mars. As Paulo Cezar Carrijo said in a news release on July 25, 2018,

"A legal imbroglio called Justice of Work costs 5 billion dollars a year to the country. Therefore, there is no lack of money to do what has to be done. What is lacking is shame in the face of all of us […]. The people who polluted streams, rivers and seas with their trash are also the same as electing inept and corrupt rulers”. (https://marsemfim.com.br/baia-de-guanabara-entenda-poluicao/; accessed on March 04, 2019).

On the other hand, cargo ships in Rio de Janeiro, whose unregulated and non-oversight companies, and their careless employees, show little zeal for the encompassing environment, which is one of the world's most eutrophic ecosystems, wash their infected basements within GB water without any authority condemning this fact. Indeed, people throw sofas, mattresses, sinks and all kinds of objects into the rivers, making the bay a true cesspool. This shows the more than obvious conclusion that education, law, health, and citizenship go hand-in-hand in a socially developed nation.

GB shoreline dwellers need wise marine spatial planning done by persistent and courageous planners and their coworkers uninterrupted by the nonsense instructions of ignoramus politicians and inept bureaucrats! There are exploitable opportunities for those endowed with vivid prospection visions and willpower. Fortunately, there are people whose voices seem to win listeners little-by-little. International pressure and personal networks have contributed to mobilize not only academics and intellectuals in general, but also the population. The purpose of this article is to expand the modest ranks of those who embrace socio-environmental causes, presenting a doable proposal for GB's recovery (and, please, do not ever say there is no money for the tasks needed!) [8]. The Petrochemical Complex of Duque de Caxias ought to be held wholly responsible, in perpetuity, for the long-term health maintenance of the mangrove region situated on the west side of the southward-facing GB. The upper estuary nearby is targeted by local fisheries of diadromous species.

2. Oceanographic Report

GB is in fact a fan-shaped lagoon in *cul-de-sac* (Figure 1). It is the result of a tectonic depression formed in the Cenozoic period. There is only a relatively strait access to the open sea, which narrows water circulation. Nevertheless, the resurgence phenomenon, strongly influenced by the wind tension and by the Brazilian Current (BC), may carry South Atlantic Central Water (SACW) into the GB up to 15 km, accordingly February 2001 thermohaline indices reported by Bérgamo [7]. The fact that SACW can be advected into the interior of GB only 15 km inside is not enough to bring relevant renewal, since such waters do not reach the more distant and polluted shores of the bay to the north and northwest. Moreover, the circulation of seawater in the most remote areas is almost totally restricted to the small speeds of tidal movements in the innermost estuary.

GB covers an area of 384 km^2, in maximum measures 30 km long from north to south against 28 km from east to west, containing >100 islands and still maintaining the little that remains of the ancient mangroves that characterized most of its perimeter border of about 131 km. There are 53 sandy beaches and a 30-40 m deep central channel (see Figure 2) [8]. As hinted above, the water circulation in GB is greatly influenced by tidal currents of semi-diurnal type, with a maximum amplitude of 1.4 m. With tidal current velocity of about 0.1 m/s in the shallower interior, an entrance 1.6 km wide, and a sandbank located in this entrance, the renewal of water at the GB's interior limits becomes negligible if contrasted with the volume of organic sewage discharged per second from the contaminated rivers.

The influence of wind is considerable in GB's fresh and salt water regimes. Carvalho showed that the wind field has fundamental role in the essential hydrodynamics of GB, changing the field of velocity of the northern portion and shifting the surface elevation field, demonstrating that the environmental management of GB must obligatorily consider the interaction between all water and the various winds [9]. Rio de Janeiro's population and infrastructure has never been blown by a tropical cyclone although this may change owing to global and regional climate regime changes since the first recorded South Atlantic Ocean cyclone reached land in the State of Santa Catarina in March of 2014. Have Brazilian politicians and bureaucrats considered the catastrophic outcome of such future storminess?

3. A Socio-Environmental Conundrum

An intricate reality has offered difficult barriers to overcome in terms of environmental recovery and preservation in Brazil, notably in the State of Rio de Janeiro, perhaps the Brazilian federation unit most affected by corruption in the last 30 years. In this State, the disregard for waste management, especially solid waste, becomes evident when one overflies GB. In 2002, CONAMA Resolution 307, later amended by Resolution 348/2004, determined that the solid waste generator was responsible for its management. This determination represented an important legal framework, determining responsibilities and stipulating the segregation of waste into different classes, making their referrals for recycling or adequate final disposal mandatory. The greatest advance in terms of legislation came when the Federal Government, through Law N# 12 305/2010, instituted the National Solid Waste Policy (NSWP), by which it created the necessary instruments for Brazil to face the main environmental, economic and social problems that arise when the management of solid waste is done inappropriately [10-11]. However, the exercise of the law and of the recommendations instituted did not come close to being effective. The amount of garbage carried to GB made it impractical to fish in several locations. Typically fishing communities such as the former *Porto da Piedade*, whose fishermen are mostly descendants of slaves, are today in pervasive poverty. Only in some areas is it still possible to carry out a modest subsistence fishery (see Figure 3 in appendix).

Residential sewage dumps are the main aggressors of the GB biome. As is known, the disposal of domestic sewage in any aquatic environment causes reduction of dissolved oxygen, pH changes and turbidity, being these dumps treated or not. In addition, industrial heavy metal dumps have been reported since 1988. The many studies of the types of pollutants and their proportions present in GB are well-known, so that it is enough to emphasize here the near-absence of sanitary conditions and sewage treatment in the poorest areas around GB, thus reflected by the high infant mortality of 23.9% in some locales, compared to other areas where the infant mortality is 4% due to the effective working existence of sewage disposal systems [12]. It is also noteworthy that in 2000 the town of Tubiacanga was the community most affected by the oil spill in GB, considered the second worst environmental accident in the region, with 1.3 million liters dumped in the waters, mangroves and bay beaches. This environmental disaster, added to more recent ones involving suddenly breached waste-impounding dams belonging to rich mining companies, did not produce the national commotion that would be expected. It seems that government neglect, besides being a mark of Brazilian management, has already infiltrated the *modus vivendi* of the communities, configuring widespread popular indifference.

The most expressive GB recovery initiative was the cooperation between the Inter-American Development Bank, the Japan Bank for International Cooperation (JBIC) and the government of Rio de Janeiro State, which elaborated the Program for Remediation of Guanabara Bay (PRGB), begun in 1994. The program proved to be a fiasco, thanks to local corruption and typical discontinuity of Brazilian politics when there is a change of government. From the large set of sewage treatment plants planned, several unities were not concluded or not connected to the sewage collect/disposal system. Not even the 2016 Olympic Games left a positive legacy for society, since health and sanitary interventions were very close to the ridiculous. Although the international community has recognized the urgency of actions to effectively conserve the marine and coastal ecosystems, mainly after the Rio+20 Conference, very little has been done from the practical point of view. “The Future We Want” document, from Rio+20 Conference, shall remain pure exercise of rhetoric if mankind does not seriously begin to think as a species, radically changing the current market model and combating the harmful effects of economic globalization. Until slightly more than a decade ago (that is, pre-2007), official maps omitted the shantytowns (slums called “favelas”)!

In short, characterized as a real territory of social exclusion, the GB is becoming the scene of an environmental devastation increasingly more difficult to reverse, especially in face of the apathy of the majority of the populace and the irresponsibility of the public power. Looking at the Brazilian reality, since it is a rich country, although burdened with one of the worst distributions of monetary wealth in the world, one should seek alignment with the UNEP Strategic Directions (2017-2020), and the Regional Seas Conventions and Action Plans, concentrating investments in the ecological protection of the coastal environments, guaranteeing quality of life and social development for those who live there.

In future, owing to anticipated future global sea-level rise, there might occur a 40 m retreat — a migration inland — of Rio de Janeiro’s famed beaches, meaning the *calçadão*, the bike-lane and the twin paths of sea-fronting Avenida will disappear beneath the high-tides. In other words, the remake of the 1984 movie ***Blame it on Rio*** will have to build a suitable new set for the eroded and submerged natural beach where the film-stars previously paraded[13]!

4. Technological Prospects

Several studies have been produced on GB, motivated mainly by the current situation of environmental degradation [14]. Such studies generally point to the more traditional measures of long-term solution since usually their authors are not macroproject minded! In fact, the conventionally designed depollution process, based only on a network of treatment plants, shall take a long time to show satisfactory results owing to the extensive replumbing of a large metropolitan region; there shall be a need for comprehensive educational programs to change the ingrained bad sanitary habits of the population, as well as vigorous enforcement measures regarding ship and industry evictions in GB. It is a very time-consuming task, too long to hope for any improvement for seaside communities to take place, considering that there is almost no truly effective environmental management in Brazil at the present time. This perception led us to conceive a macrosystem of pipelines transporting oceanic water under pressure to the generally stagnant shallow northern area of the GB, creating a suitable piped artificial current capable of accelerating, when it exits the pipe, the massive renewal of the seawaters and promoting a more immediate bubbled oxygenation for the reactivation of the artisanal fishery, bringing long-term economic relief to the upper estuary fishing communities.

Nowadays, large-scale engineering interventions to divert ocean water within intra-continental water bodies are not yet common actions, although there is a growing emphasis on inter-basin water transfer megaprojects for environmental, economic and social purposes because of noticeable climate regime change. Our megaproject is addressed to all of these purposes, with the reminder that ten million people today are affected directly or indirectly by GB's deplorable conditions. Of course, it must work with other devices and long-term measures. However, in Latin America one has to take great care to make things happen as they should be, since socio-political history of that world region’s countries is not among the most encouraging. For instance, Rodrigues *et al.* applied an interesting historical classification of water legal treatment in

Brazil [15], in which three distinct phases are identified as 1) the navigability phase, 2) the hydroelectricity phase, and 3) the environmental phase (the current phase). We note that in all of them management mistakes were evident and have been recognized at least by the Brazilian engineering community. In most cases, mistakes are made by neglecting the negative effects of geographically large-scale projects [16], which are announced only on the side of the benefits that unfortunately serve the dominant minorities more than civilians who need broad public policies. The poorest populations are those most affected by the negative impacts (expropriation, environmental pollution, etc.).

That is not the case with our proposal. The benefits of our presently defined macroproject to accelerate GB's seawater quality recovery are almost immediate for both fishing and tourism, and it could bring back the profitable activities that would occupy a large part of today's idle local working class. Here, it is worth noting that the Ipanema Submarine Outfall, first installed by 1974, still releases untreated waste-water that during flood tide enters GB! The periods of discharge of oceanic water would be articulated with the tidal cycles in order to counter-balance the acceptable levels of the GB seawater condition. The forceful current induced by the pipelines would act in anti-clockwise flow from the northeast shore of the GB, benefiting the most critical areas and forcing oxygenation in *Fundão* channel and *Praia de Ramos*. The favelas-bordered *Fundão* channel, an artificial mini-estuary created between 1949-1952 by the linking of eight islands with landfill, is characterized by oil and sewage-polluted seawater and mildly radioactive sediment. *Fundão* Island is home to *Cidade Universitária* campus and the Rio Science Park since 2003. Solid waste containment barriers, even simple racks, placed at river mouths would prevent garbage from flowing into the ocean, while an effective system of selective collection and recycling would direct the accumulated debris to proper disposal. As in Tokyo, it is possible to add polyester fiber screens in three layers for E. coli filtration in the estuarine zones. Strategically positioned biological stations would monitor GB's ecological dynamics, making periodic measurements on water quality and indicators related to the activity of micro-organisms, fauna and flora in general. Can we, someday in the near-future, expect technical counseling from technically knowledgeable mentors working, since 2015, at the *Museo do Amanhã* (Museum of Tomorrow)?

One negative aspect, however, should be studied cautiously. GB has a hybrid bed, partly consisting of mud in the inner mangroves and variable sizing sand on the banks of the islands and on the edges closer to the ocean. Thus, the transfer of ocean water should take into account a filtering process that prevents the traffic of large amounts of sand, so that the balance of mangroves is preserved. Over time, with the set of sewage treatment plants in operation, the stabilization of the freshwater/saltwater mix shall occur naturally. Only then shall we be able to await the results of education in the new generations of young people, certainly more engaged in questions of humanity's survival on this beautiful and mistreated planet.

Acknowledgements

This article was elaborated within the Postgraduate Program in **Inclusion and Social Development of UNIDESC - University Center**. The authors thank Tânia Rossi from UNIDESC for the support and encouragement of this research.

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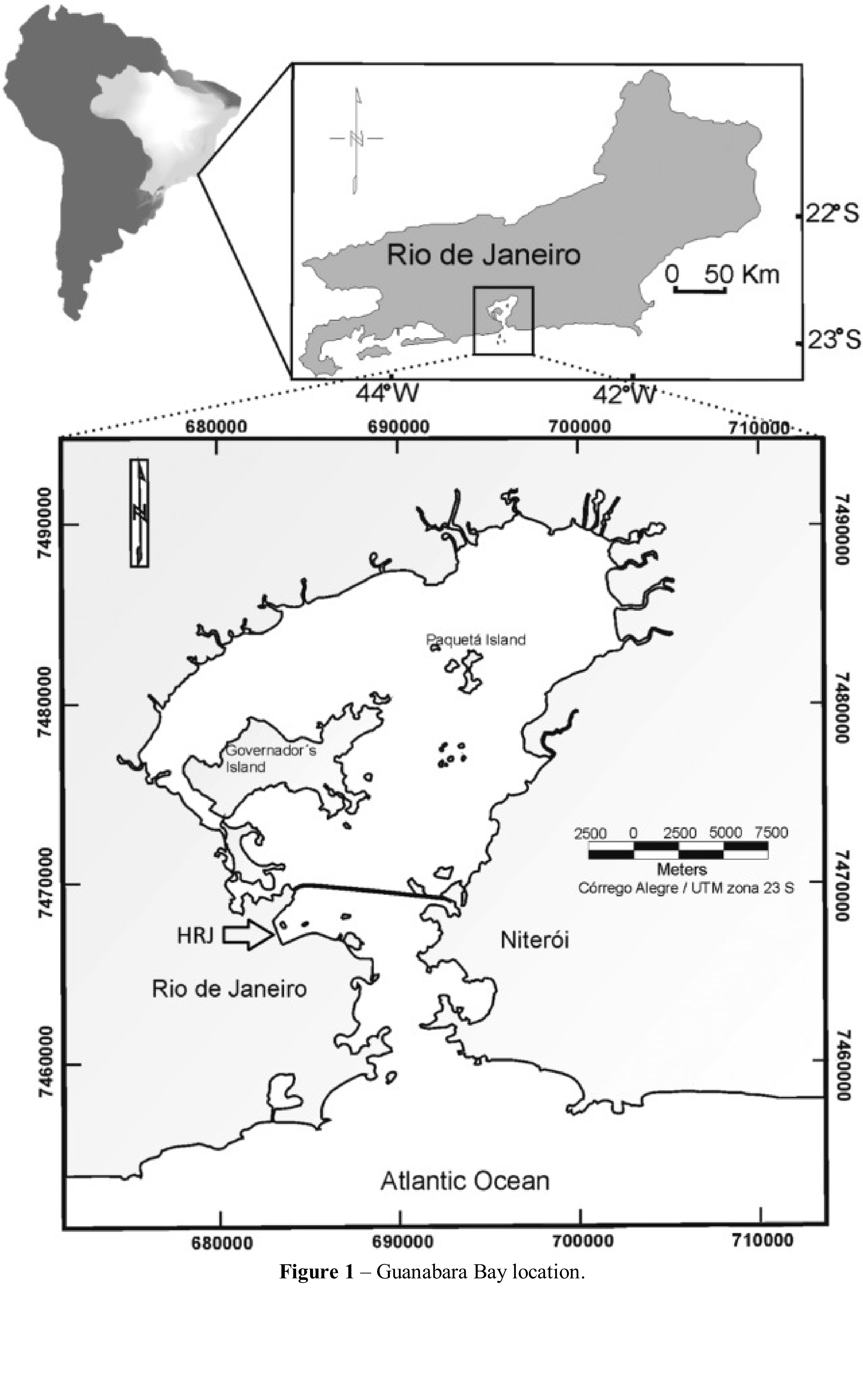

Figure 1 – Guanabara Bay location.

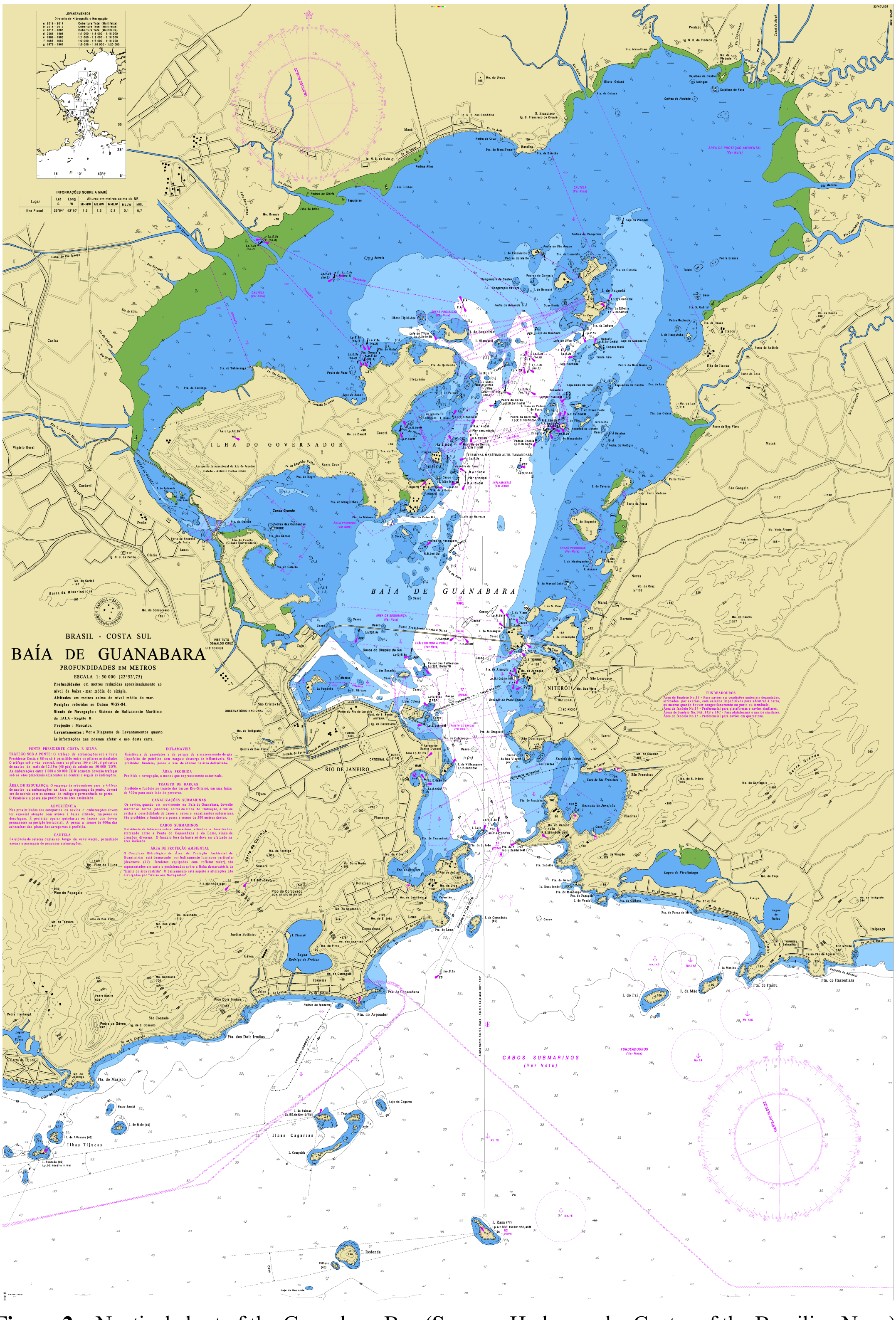

Figure 2 – Nautical chart of the Guanabara Bay (**Source**: Hydrography Center of the Brazilian Navy).

Appendix – *Mauá* Beach's selected portraits: traces of paradise






Figure 3 – Although unsuitable for bathing, *Mauá*'s beach, or *Guia de Pacobaíba*, still houses fishermen; friendly people, hopeful for a new beginning among the herons (Copyright ® 2019, Serpa and Cathcart).

Revista Brasiliense de Engenharia e Física Aplicada

21st Century Macro-Imagineering

Lake Titicaca Hydropower Megaproject

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Received: 12/08/18/ Accepted: 12/18/18/ Published: 04/30/19.

Abstract: Territorially shared by Peru and Bolivia, South America's largest freshwater lake is navigable Lake Titicaca, situated on an highland endorheic river basin of the Andes Mountains. Currently, Lake Titicaca is mostly regulated by ever-changing Nature. But, since its closure during 2001 AD, a small anthropogenic dam (at elevation 3804 m), emplaced at the headwaters, the Rio Desaguadero is still the altiplano lake's only flowing freshwater outlet. Intriguingly, from circa 1908 AD, Macro-Imagineers foresaw the creation of a second, completely artificial, flowing freshwater outlet for Lake Titicaca's 935 km3 of valuable liquid freshwater accumulation. Such a lake-tapping hydropower megaproject could allow diverted freshwater to perhaps reach the Pacific Ocean. Here, we cursorily reappraise several similar 20th Century dam and pipeline macroproject proposals, with the educative goal, achieved by careful reconsideration, of exposing the basic megaproject proposal physics related to a potential major future South America hydropower installation.

Key words: hydropower, Macro-Imagineering, Lake Titicaca.

Resumo: Territorialmente compartilhado pelo Peru e pela Bolívia, o maior lago de água doce da América do Sul, conhecido como Lago Titicaca, está situado em uma bacia endorrêica do altiplano da Cordilheira dos Andes. Atualmente, o Lago Titicaca ainda é regulado principalmente pela natureza local em constante mudança. Mas, após a construção de uma pequena barragem em 2001 d.C. (a uma altitude de 3804 m) situada nas cabeceiras, o rio Desaguadero permanece o único escoadouro natural do lago. Curiosamente, por volta de 1908 d.C., os macro-imaginadores previam a criação de uma segunda saída, completamente artificial, para os 935 km^3 da valiosa água doce líquida. Semelhante megaprojeto hidrelétrico poderia permitir que a água doce desviada alcançasse talvez o Oceano Pacífico. Aqui, reavaliamos várias propostas semelhantes de macroprojetos de barragens e dutos do século XX, com o objetivo educativo, guiado por uma cuidadosa avaliação, de expor a física básica da proposta para uma futura instalação hidrelétrica na América do Sul.

Palavras-chave: hidrelétrica, Macro-Imaginação, Lago Titicaca.

1. Introduction

Today's documented data and information — knowledge — is fraught with unlinked puzzle pieces caused by knowledge-base gaps as well as truly unknown errors of which humans are, as yet, absolutely unaware. Macro-Imagineering's adherents work against time-schedules founded on the limitations of well-defined monetary budgets. Because the hydro-social cycle is very complicated in any region anywhere within Earth's bioshell, no macro-imagineer is likely ever to be able to examine every elemental question to the exacting degree that prudent geoscientific rigor would usually demand; indeed, the wisdom of "Macro-Imagineering Judgement" typifies this constrained capability, as well as necessary outcome, to arrive at some timely and good work-to-be-done decision with whatever geoscientific data and information are available. Sometimes cost-escalating over-design of megaprojects located at especially difficult real-world worksites results [1]. Nature sculpted Lake Titicaca's tectonic uplift plain Basin — also known as "watershed" — as an ecological system or geographical unit located at 150 45' South latitude by 690 25' West longitude. The present-day and projected future hydro-social cycle driver of cooperation between Peru and Bolivia could be increasingly clearer climate regime change risks. Bolivia and Peru first commenced organized co-management of Lake Titicaca during 1955, sharing bathymetry data since 1976 and comprehensive hydro-meteorological information by 1985. Subsequently, Peru and Bolivia finalized the Lake

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Titicaca, Desaguadero River, Poopo Lake and Coipasa "Salar" Binational Master Plan (TDPS system) after the negotiations lasting from 1955 until 1996 AD [2]. Figure 1, below. Placement of an outline map of territory at the beginning of any proposed macroproject general description indicates a substantial intellectual adventure will follow.

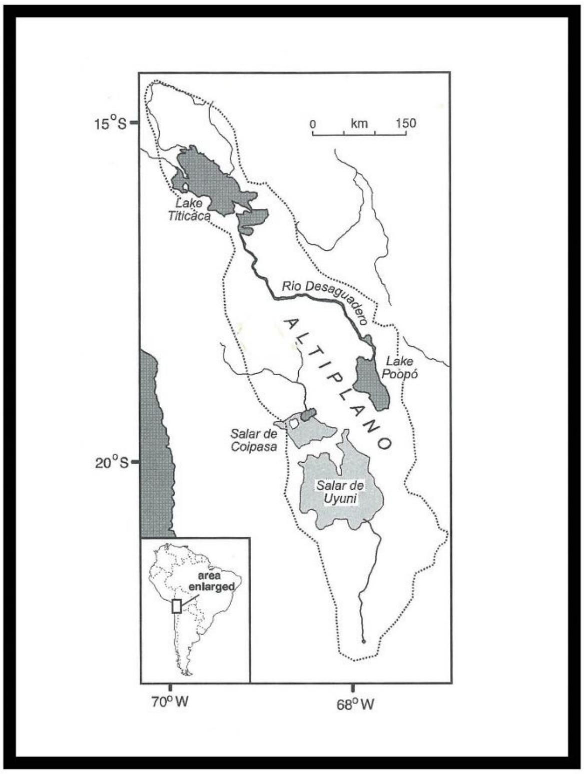

Figure 1. Map of TDPS.

Climate change risks, with all the implications for the mandates of the TDPS system governing authorities, remain difficult to forecast, but could be severe, especially when superincumbent upon existing macro-problems (increasing population density and sewage flows, variability of supply and quality of potable water, increased urbanism and agricultural consumptive demand for irrigation water, and legal conflicts concerning end-usage) [3]. Bad freshwater management practices and procedures, exemplified most tellingly by the rapid area diminishment of the present-day hydrologic condition of the Aral Sea in the Old World [4], are to be avoided in the New World! Although Peru has the greatest concentration of extant glaciers in our world's Tropic Zone, the glaciers essential to maintaining Lake Titicaca's mass are receding by sublimation, combined with a shorter precipitation season, already measured and computer climate model-predicted air warming and increased solar radiation impacts [5], the result is an observable freshwater volume reduction of Lake Titicaca and, consequently, also its total evaporative area. During the 20th Century Lake Titicaca fluctuated ~7 m [6]. The average precipitation directly onto Lake Titicaca is ~800 mm/year with the mean annual evaporation estimated to be ~1,700 mm/year [7]; the endorheic basin in which the variably-sized Lake Titicaca pools is ~56,494 km^2 — approximately 39.1% of the TDPS. By international agreement Peru retains authority over 74.1% of the Lake Titicaca Basin section of the TDPS, Bolivia holds absolute responsibility for 25.9%.

2. Geographical particulars and specifics

The Andes Cordillera arose because of the interaction of the South America tectonic plate and the Nazca tectonic plates [8]; this subduction zone is our world's oldest and lengthiest. Located on a tectonic uplifted plain, the ancient Lake Titicaca is probably a Quaternary Period remnant after its Basin filled to a maximum ~4,000 meters elevation. Nowadays, the tourist-appreciated scenic 8,500 km^2 Lake Titicaca has an estimated surface elevation of ~3,810 m; human altitude sickness symptoms ordinarily present themselves above 2,500 m altitude and yet permanent human occupation of the highlands by roaming hunter-gatherers occurred sometime before 5000 BC [9]! Set in the Tropic Zone where zonal overturning of the Walker circulation occurs, the TDPS is known to have been beset with mega-drought periods as well as pluvial excess periods such as the Last Glacial Maximum, some of which have been experienced by people living within Lake Titicaca's watershed.

Indeed, the presently existing elevation of the lake's watery free-surface has submerged ancient artifacts and settlements. The pattern of verticality of the landscape's anthropogenic components — terraces, raised farm fields, sunken gardens and manicured pastures — derives from climatic and biotic differences directly related to height above sea-level. Freshwater availability in Peru varies markedly: the Pacific Ocean coastal region is home to 65% of Peru's 32.5 million people but contains only ~1.8% of the nation's water resources whilst TDPS has but 5% of the country's population. Peru's upper-Amazon River Basin zone, however, produces 95% of the freshwater resources! So, not so surprisingly, Peru has attempted to satisfy freshwater consumptive demand of its coastal cities with a number of geographically large-scale macroprojects consisting of upper-Amazon River Basin river diversions/reversals. The rivers flowing down the steep western slope of the Andes Mountains separating Lake Titicaca Basin from the Pacific Ocean-facing coastal region have small watersheds and only those which drain still glaciated mountains have sufficient freshwater for continuous irrigation agriculture.

Commonly it is assumed that the greatest potential for new hydropower generation is from unregulated rivers flowing down the steep eastern slopes of the Andes Mountains. Peru has an urgent requirement to increase its renewable sources of electrical energy because human population increase, particularly in the Lima-Callao region, imposes an increasing public electricity demand (supposedly, the theoretical estimated hydropower potential for all Peru is ~206,107 MW; as of AD 2017, the installed hydropower capacity totaled ~5,385 MW).

Lake Titicaca constitutes a freshwater resource for approximately 2.5-3.0 million persons residing and working in Peru and Bolivia. However, yearly ~90% of the lake's pooled freshwater is lost from the Basin into the thin air through technically unmitigated evaporation whilst ~10% is allowed to leave Lake Titicaca as a managed freshwater discharge through the Rio Desaguadero Dam. Figure 2, below. "Proyect Especial del Lago Titicaca" — the PELT project of 1989-1993 AD — provided the necessary and vital TDPS hydrological data that eventually led to an international agreement-defined freshwater flow rate of 20 m3/second to be equally shared by Peru and Bolivia downstream of the Desaguadero Dam and a nearby International Bridge [10].

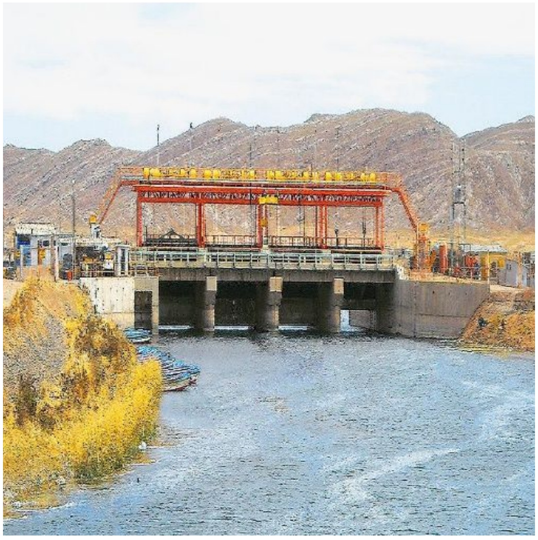

Figure 2. Flowing river freshwater-level control gates at present-day undersized Desaguadero Dam located at the shallowest, slightly polluted southern end of Lake Titicaca. We postulate the dam could be heightened and extended horizontally with freshwater-restraining flanking earthen embankments to increase massively upstream freshwater retention capacity of Lake Titicaca.

3. Stabilized Lake Titicaca hydropower serving some coastal Peruvian communities

Whilst work on the "National Map of Peru" commenced on 10 May 1921 by plane-table and alidade methods, reportedly it remains incomplete even well into the age of the satellite-centered Global

Positioning System [11]. Accurate topographic mapping is essential to the macro-planning of any megaproject that relies on freshwater's falling gravitationally a long distance from the 4,000-4,200 m-high western heights of the breath-taking Andes Mountains to the Pacific Ocean's shoreline! Charles Reginald Enock (1868-1970), a renowned British explorer of Peru, was the first to explore afoot and impressionistically chart the rough topography and valuable surface mineral resources of the TDPS as well as its immediately surrounding mountainous terrain [12]. He suggested a 120 km-long tunnel might convey Lake Titicaca's excess fluid content to the populated coastal region adjacent to the Pacific Ocean and generate hydropower simultaneously [13]. (NOTE: electricity-generating hydropower's global history started after AD 1880.) A vertical drop to AD 2018 prevailing sea-level of, say, 20 m3/second from an selected elevation of 3,810 m could, theoretically, produce 640 MW, nearly 11.88% of Peru's current actual electricity production or 0.31% of its theoretical maximum hydropower electricity production. Although we cannot be certain, we suspect C.R. Enock may have been inspired by the USA macro-imagineer Alexis Von Schmidt (1821-1906) who, from 1865 AD onwards, proposed and promoted a freshwater aqueduct to deliver at ~23 m3/second to the City and County of San Francisco originating at, and drawing from, the famed Lake Tahoe [14]; Lake Tahoe straddles the State of California-State of Nevada boundary—therefore it is also an enormous bistate managed pool of freshwater. Whatever is the verifiable historical truth, the idea was again bruited during the late-1950s by French macro-imagineer Marcel Mary [15]. Translated into summary English and generalized, Mary offers the opinion that a diversion of Lake Titicaca to the Pacific Ocean by sediment and rock piercing at great ground depth would present operators with a large hydraulic head — perhaps as much as 3,500 m — and could supply ample irrigation liquid to Peru's coastal farmers; if Lake Titicaca were artificially drained by valve-controlled pipe at its deepest lake-bottom location (~284 m near Soto Island north of Puno), freshwater could be made to slant-fall ~3,500 m, ultimately to join a river which releases flowing freshwater into the Pacific Ocean. Within a range of about 250-700 m, both Francis and impulse turbines can be utilized for electricity generation, possibly in multiple power-stations (output and dynamic behavior of serial water-turbines in a hydraulically coupled system) that could then feed electricity to long-distance transmission-lines draped across the rugged landscape of the upper slope of the western Andes Mountains.

4. Proposal physics: assumed base-of-mountain hydropower station

Rock-mass conditions that influence tunneling costs the significantly are mechanical rock properties, discontinuities in rock-masses and the presence or absence of suddenly interfering hot or cold groundwater inflows. As a linear excavation operation, tunneling has special quirks in access and logistics in the overcoming of undetected and unforeseen macro-problems; there is a unique interdependence between design and construction. Selection of the best course of any Tunnel Boring Machine (TBM)-dug freshwater-tunnel ought to be done on the basis of on-site Macro-Engineering assessment bolstered by astute geological surveys and studies. Studies will aim to predict, in the alternative suggested freshwater-tunnel routing, the influence of the rock mass conditions and the extent of definitive field-studies that necessarily must be done thoroughly and correctly before commencement of expensive water-tunnel digging and lining tasks can be undertaken. The economic benefits following such crafted studies can be estimated as a considerable percentage of the final construction cost of tunnel driving using a suitable TBM. As with other modern-day macroprojects, applied Macro-Engineering has changed from its 20th Century incarnation; 21st Century applied Macro-Engineering leaders of any megaproject today must have a list of titled cultural groups to be met, Environmental Impact Statements to be submitted for approval with various governing organizations, national and international laws to be respectfully complied with, and regional and even world public hydro-social cycle concerns to be addressed. Yet the result — a legal and financial "Go-ahead!" — if done properly, is well worth these constraints of time and direct financial hardship: hydropower technology chosen wisely, democratically, and consensually, rather than being dictated. In Europe, Switzerland enhanced its national identity via tunneling macroprojects [16] and, during the 21st Century, other ecosystem-nations of which Europe is comprised will do the same [17-18]; in other words, Peru-Bolivia could have a big emerging opportunity to cooperatively buff their respective international reputations in several disciplines of modern Technology! The only comparable tunneling megaproject being contemplated currently is sited in China's Tibet [19-20]; China's scheme involves a 42 km-long inclined headrace tunnel.

Ordinarily, Lake Titicaca hydropower potential would remain untapped and worthless (on a significant geographical and economical scale) in the near-term future for a number of logical related reasons: (I) the absence of Environmental Impacts Statements; (II) formidable geological and geomorphological impediments such as infamously powerful earthquakes and jagged incidental terrain; (III) nearly non-existent traffic infrastructure such as tunneled highways and railways; (IV) high-to-very-high initial monetary investment costs and long-period financial pay-backs; (V) the requirement for reliable long-distance weather-proofed electricity transmission lines; (VI) the reluctance of international financiers to consider low-interest loans to Peru and Bolivia; (VII) occasionally volatile and sometimes inconsistent national political opinions regarding priorities of national, regional and centralized or decentralized energy system development and (VIII) the considerable on-going development of alternative renewable energy resources. Furthermore, we must assume that future global climate regime change, expressed regionally, may instigate flexibility requirements for many existing and planned infrastructures. If, for example, the altiplano climate regime becomes drier than today's, then Lake Titicaca will be gradually reducing in area; on the other hand, if the altiplano becomes climatically wetter, Lake Titicaca will increase in volume and may then pose a severe emplaced lake-shore infrastructure damage risk. Our offered technically-based macroproject proposal might create new facts on the ground — that is, a situation is formed whereby, in either instance, Peru's coastal population will flourish and prosper with the availability of additional hydropower!

The conventional method to harness hydropower obtained from a high-elevation permanent lake is by drilling an admittedly difficult-to-complete inclined tunnel through the intermediate hard-rock mountains. But, in our case, this usual method is excessively expensive and requires inordinately long periods of dangerous excavation. So far, the world-practice of tunneling has no experience with very long-distance slant-bored tunnels, especially those incised by TBM in hard-rock mountain geological formations. As of 2018 AD, we offer an important technical innovation: to install a hermetic steel or prefabricated reinforced-concrete tube — possibly similar to that which might be used by a Hyperloop installation — emplaced by heavy-life helicopters, possibly entirely robotic in few critical operations, over the directly affected landscape of the western Andes Mountains (Figure 3).

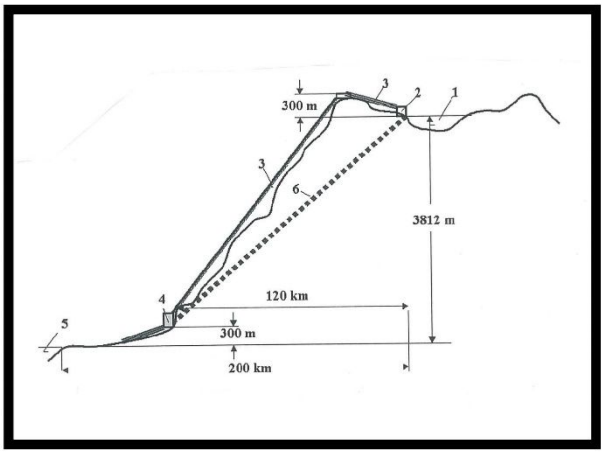

Figure 3. Sketch of proposed Lake Titicaca Electricity Generation Station. Notations: 1= Lake Titicaca; 2=freshwater pumping plant; 3= water tube (sub-aerial pipeline); 4= hydropower station; 5= Pacific Ocean; 6= possible subterranean tunnel through rocky mountain. In Peru, the shortest distance is between Puno to nearby mountain top is a distance of ~20 km, the linear distance to the nearest natural flowing river west of the penetrated mountain that drains into the Pacific Ocean is ~60 km and, thence, to the Pacific Ocean is ~200 km. In Bolivia, the shortest distance from Pucarani to Rio Zongo, after passing the ridge, is ~50 km.

4. Macro-Imagineering conclusions

The inexpensive hydropower plant, having a generation capacity of 640-1000 MW may be built during the 21st Century near Lake Titicaca without any hazardous, ugly modification to Lake Titicaca's picturesque region or its visible volume of freshwater with a freshwater expense of ~20-35 m3/second. For this result to occur, the floodgates of the Rio Desaguadero Dam must be, at minimum, permanently shut after international negotiations have been successfully concluded between Peru and Bolivia. If we choose to harness permanently a flow of freshwater moving at >30 m^3/second then Lake Titicaca's free-surface area will assuredly decrease. We can, however, generate, transmit for distribution and end-use, more or less for-ever, a maximum power output of up to 6,100 MW. Lake Titicaca will vanish, but our innovative recommendation will still produce utilizable electricity. If, sometime, we find a truly economical means for halting or greatly reducing the high-altitude lake's freshwater evaporation — as, for instance, by installation of particularized floating covers of huge numbers of plastic shade-balls [21] — then it will certainly become possible then to "Save Lake Titicaca" in perpetuity whilst also obtaining a great quantity of exportable generated electrical power! Produced electricity will be fed into the dispersed main load-centers by the national grid whilst the released freshwater can be used for downstream agriculture pursuits and to fulfil urban population needs. The most likely geopolitical obstacle: Bolivia may never agree to such freshwater diversion because Lake Titicaca is already a shared resource managed at the present time by the TPDS. However, western Bolivia also has a known need for low-cost electricity and, therefore, we think that an amicable binding international agreement is potentially possible.

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Revista Brasiliense de Engenharia e Física Aplicada

Lenteamento Gravitacional, Grupo Local e Estrutura da Galáxia: Uma Revisão

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Received: _02/25/19_ / Accepted: _04/11/19_ / Published:_ 04/30/19_.

Abstract: The subject of "gravitational lensing" has gained expression since the 1960s. Over the last twenty years, with the dizzying advances in information technology and space remote exploration technology, numerous simulations and observational results in this field have produced significant changes in our way of seeing the universe both on a large scale and within the limits of the most accessible galactic neighborhood. In the present study, while focusing on the effects of weak gravitational lenses and microlensing in the Local Group, I will take a brief but solid approach to the basic theory of gravitational lenses, referring the reader, whenever necessary, to the world scientific literature.

Key words: Gravitational lenses, Local Group, week lensing.

Resumo: O assunto das "lentes gravitacionais" ganhou expressão desde a década de 1960. Nos últimos vinte anos, com os avanços vertiginosos da tecnologia da informação e da tecnologia de exploração espacial remota, numerosas simulações e resultados observacionais neste campo produziram mudanças significativas em nossa maneira de ver o universo, tanto em grande escala quanto dentro dos limites da vizinhança galáctica acessível. No presente estudo, embora me concentre nos efeitos das lentes gravitacionais fracas e do microlenteamento no Grupo Local, farei uma breve, porém sólida, abordagem à teoria básica das lentes gravitacionais, remetendo o leitor, sempre que necessário, à literatura científica mundial.

Palavras-chave: Lentes gravitacionais, Grupo Local, lenteamento fraco.

Letras romanas

J: jacobiano
G: constante gravitacional

Letras gregas

ξ : parâmetro de impacto
α : ângulo de deflexão
μ : magnificação
θ_E : raio de Einstein

1. Introdução

O assunto "lentes gravitacionais" ganhou força a partir dos anos 60. Nos últimos trinta anos, com os avanços da informática e da tecnologia de prospecção espacial, numerosos trabalhos com simulações e resultados observacionais nesse campo têm produzido mudanças expressivas em nossa maneira de ver o universo, tanto em larga escala quanto nos limites da vizinhança galáctica mais acessível.

A busca por efeitos de lenteamento na vizinhança da Via-Láctea compreende um considerável capítulo do estudo geral sobre lenteamento gravitacional,

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provavelmente capaz de ocupar uma vida de pesquisa sem que seja completamente esgotado. Isso se deve em parte a que muito tempo de exposição é necessário para computar as possibilidades de lenteamento entre um observador na Terra e as estrelas mais importantes, por exemplo, da Grande Nuvem de Magalhães. Além disso, a composição do halo por si só já constitui um tema com nuances próprias de grande incerteza, sobretudo no tocante à composição da matéria escura. Finalmente, as dificuldades inerentes à tarefa de observar impõem as costumeiras restringências de precisão, algo com que sempre teremos de nos preocupar em qualquer modelagem, e que nos faz repensar com frequência nossos métodos e objetivos.

A potencialidade para o esclarecimento detida pela teoria do lenteamento gravitacional compreende desde a descoberta de planetas até as possíveis implicações sobre a radiação cósmica de fundo. No presente estudo, embora me concentre nos efeitos de lenteamento gravitacional restritos ao grupo local, farei uma abordagem resumida, porém sólida, da teoria básica das lentes gravitacionais, remetendo o leitor, sempre que necessário, às referências consagradas na literatura científica mundial.

1.1 O estado da arte

De acordo com a teoria geral da relatividade, a gravidade nada mais é do que a realização da curvatura do espaço-tempo [4]. Consequentemente, os raios de luz podem ser curvados na vizinhança de um objeto massivo. Ainda que o primeiro cálculo conhecido acerca da deflexão de um raio de luz por um objeto massivo tenha sido realizado por Soldner em 1801 [17] com base na mecânica Newtoniana[1], a busca efetiva por lentes gravitacionais remonta a princípios do século XX (Eddington 1919, primeiro a testar as previsões de Einstein, e Lodge 1919), sendo Zwicky (1937) quem se antecipou na compreensão do efeito de lenteamento em sua dimensão cosmológica [17]. Em 1964, Sjur Refsdal apresenta pela primeira vez um método para determinação da massa de uma estrela atuando como lente [17].

Apesar dos diversos artigos publicados durante os anos 80, somente em Blandfod & Narayan (1992) encontramos uma abordagem mais concisa e sistemática do assunto, resumindo quase vinte anos de estudos (Dyer & Roeder 1973, Mitrofanov 1981, Narayan *et al* 1984, Crawford *et al* 1986, Blanchard & Schneider 1987, Sasaki 1989, Watanabe & Tomita 1991, Bartelmann & Schneider 1991) [16]. No mesmo ano aparece o primeiro livro dedicado exclusivamente ao tema (Schneider *et al* 1992) [17]. Em particular, Chang & Refsdal (1979) e Gott (1981) já haviam observado que mesmo uma imagem dupla não resolvida de um quasar, criada por uma massa pontual no halo de uma galáxia distante, pode ser analisada em termos do tempo de variação do brilho combinado das duas imagens [16]. Dessa maneira, é possível detectar os efeitos da matéria escura bariônica na forma de anãs marrons e objetos jovianos.

A partir de 1992 sucederam-se numerosas contribuições ao estudo das lentes gravitacionais [2, 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Dos trabalhos mais recentes restringir-me-ei a mencionar a importante compilação de Merten (2010) [8].

1.2 Dos objetivos do artigo

O conceito de lente gravitacional, em sua acepção mais ampla, resume um coletivo de efeitos do campo gravitacional sobre a radiação eletromagnética, assumindo-a geometricamente representada por raios. Minha pesquisa é baseada sobretudo no lenteamento gravitacional fraco, acerca do qual apresentarei os aspectos teóricos mais relevantes com o auxílio de algumas simulações computacionais. Em seguida, farei

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[1] Soldner determinou que o ângulo de deflexão no limbo solar seria de 0.″84, metade do valor calculado pela teoria geral da relatividade.

uma revisão sucinta sobre o Grupo Local, sua estrutura e sua composição. Por último, discutirei o que de melhor se tem produzido no sentido da aplicação do conhecimento acumulado sobre lentes gravitacionais ao estudo da estrutura da galáxia, partindo da ocorrência de lenteamento no Grupo Local.

2. Teoria básica das lentes gravitacionais

Admitamos que o halo da nossa galáxia seja composto, dentre outros objetos, por uma quantidade não negligenciável de astros escuros massivos. Se algum desses objetos do halo estiver próximo o bastante da linha de visada de um observador focado numa estrela distante ocorrerá um efeito caracterizado pelo abrilhantamento temporário da luz proveniente da estrela em foco. Tal efeito, descrito com mais detalhes adiante, é denominado microlenteamento (*microlensing*).

Na maioria dos casos de interesse em astrofísica é suficiente usar o campo gravitacional no limite fraco para a descrição do efeito de lenteamento próximo a massas pontuais. Consideremos o senário descrito na Figura 1. Assumindo simetria esférica, com a massa pontual no centro das coordenadas, qualquer trajetória através do plano da lente desde a fonte até o observador O será defletida (veja detalhe na Figura 1.a). Uma vez que o sistema de coordenadas é arbitrário, podemos adotar o plano $\theta = \pi / 2$ para o caminho da luz.

Tendo em mente o esquema da Figura 1, podemos escrever a equação da lente,

$$\beta = \theta - \alpha, \tag{1}$$

onde

$$\alpha = \left(\frac{D_{ls}}{D_{os}} \right) \hat{\alpha}$$

é o ângulo de deflexão.

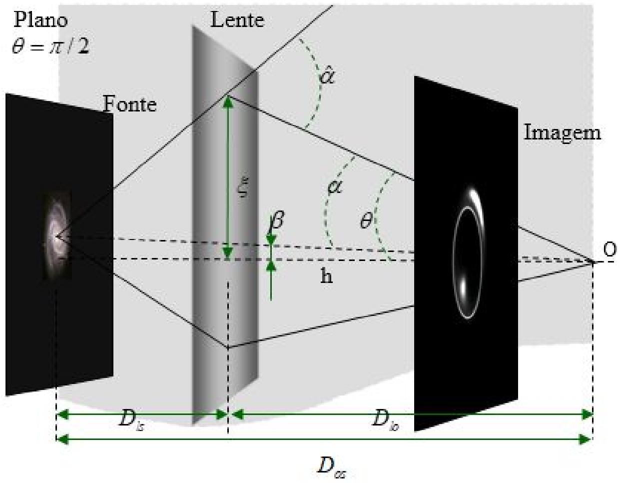

Figura 1 - Esquema típico de uma lente gravitacional simples.

Dado que

$$\hat{\alpha} = \frac{4GM}{c^2 \xi}$$

e

$$\xi = D_{lo} \theta \quad \text{(para ângulos pequenos)},$$

sendo ξ o "parâmetro de impacto", é lícito reescrever a equação (1) como

$$\beta = \theta - \left(\frac{D_{ls}}{D_{os} D_{lo}} \right) \frac{4GM}{c^2 \theta}.$$

Definimos o chamado raio de Einstein pela igualdade

$$\theta_E = \sqrt{\frac{D_{ls}}{D_{os} D_{lo}} \frac{4GM}{c^2}} \therefore$$

$$\beta = \theta - \frac{\theta_E^2}{\theta} \quad \therefore \tag{1.a}$$

$$\beta = \frac{\theta^2 - \theta_E^2}{\theta}.$$

Finalmente,

$$\frac{\theta}{\beta} = \frac{\theta^2}{\theta^2 - \theta_E^2}.$$

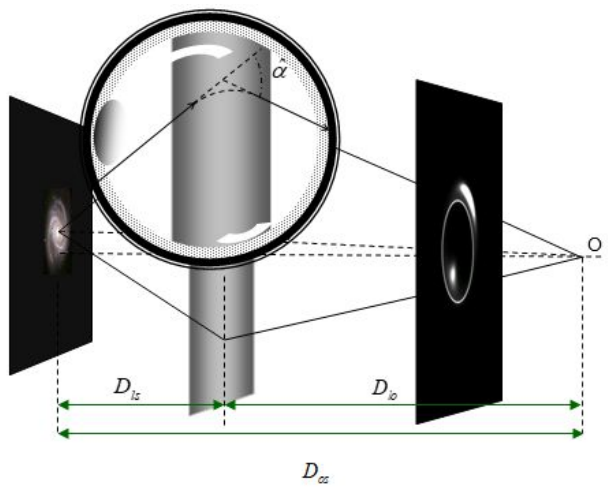

Figura 1.a- Detalhe do caminho real percorrido por um fóton cuja trajetória sofre deflexão por lenteamento gravitacional.

A expressão (1.a) é uma simples equação de segundo grau com soluções

$$\theta_{1,2} = 1/2\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right),$$

as quais fornecem as posições das duas imagens geradas por uma fonte pontual.

Chegamos assim à magnificação $\mu_{1,2}$, a soma das magnificações das imagens individuais dada em função do raio de Einstein θ_E,

$$\mu_{1,2} = \left(1 - \left[\frac{\theta_E}{\theta_{1,2}}\right]^4\right)^{-1} = \frac{u^2+2}{2u\sqrt{u^2+4}} \pm \frac{1}{2}, \qquad (2)$$

em que u é outro parâmetro de impacto definido como a separação angular entre fonte e lente medida em unidades de θ_E segundo

$$u = \beta / \theta_E.$$

Podemos chegar à expressão da magnificação fazendo $y = \beta / \theta_E$ e $x = \theta / \theta_E$, donde

$$y = x - \frac{1}{x}.$$

A magnificação produzida por qualquer lente axialmente simétrica é dada pela Jacobiana

$$\mu = \frac{y}{x}\frac{\partial y}{\partial x} = \left(1 - \frac{\alpha}{x}\right)\left(1 - \frac{\partial \alpha}{\partial x}\right)$$

$$= \left(1 - \frac{1}{x^2}\right)\left(1 + \frac{1}{x^2}\right)$$

$$= 1 - \left(\frac{1}{x}\right)^4 \therefore$$

$$\mu = \left[1 - \left(\frac{1}{x}\right)^4\right]^{-1}.$$

Compreendendo melhor o conceito de magnificação, lembremos que a quantidade de luz recebida de uma estrela é determinada pelo ângulo sólido subentendido pela estrela. O ângulo sólido por sua vez é compreendido pela área angular aparente da estrela no céu. O efeito de lenteamento aumenta o ângulo sólido sob o qual nós recebemos a luz, aumentando assim a quantidade dessa luz recebida.

Dessa forma, se nós pudermos calcular o ângulo sólido da estrela na ausência da lente e em seguida com a lente, a magnificação seria, em última análise,

magnificação = ângulo sólido com lenteamento/ângulo sólido sem lenteamento.

Um dos mais interessantes fatos relacionados ao efeito de lenteamento é que um dos autovalores da matriz Jacobiana da equação da lente desaparece na curva crítica, isto é, sobre a curva para a qual a equação da lente é estacionária. Formalmente, a equação da lente é quadrática na direção do autovalor esvanecente ("direção crítica").

Tomemos a equação da lente na forma complexa,

$$\omega(x,y)=(x+iy)+\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)\Phi(x,y),$$

onde o potencial gravitacional bidimensional é relacionado com a densidade superficial κ (em unidades de densidade crítica) por

$$-\Delta\Phi=2\frac{\Sigma}{\Sigma_{crit}}=2\kappa.$$

Agora, introduzimos $z=x+iy$ e $\bar{z}=x-iy$ de tal modo que

$$\begin{cases}x=z-iy\\ x=\bar{z}+iy\quad(+)\end{cases}$$
$$\overline{}$$
$$2x=\bar{z}+z\ \therefore$$

$$x=\frac{1}{2}(\bar{z}+z), \qquad (3)$$

$$y=\frac{1}{2i}(z-\bar{z}). \qquad (4)$$

A diferencial total de uma função complexa $\omega(z)$ é

$$d\omega(z)=\frac{\partial\omega(z)}{\partial x}dx+\frac{\partial\omega(z)}{\partial y}dy.$$

Inserindo as imposições definidas em (29) e (30),

$$d\omega(z)=\frac{\partial\omega(z)}{\partial x}\left(\frac{\partial x}{\partial z}dz+\frac{\partial x}{\partial\bar{z}}d\bar{z}\right)+\frac{\partial\omega(z)}{\partial y}\left(\frac{\partial y}{\partial z}dz+\frac{\partial y}{\partial\bar{z}}d\bar{z}\right);$$

$$d\omega(z)=\frac{1}{2}\frac{\partial\omega(z)}{\partial x}\left(dz+d\bar{z}\right)+\frac{1}{2i}\frac{\partial\omega(z)}{\partial y}\left(dz-d\bar{z}\right).$$

Consideremos as derivadas de Wirtinger de uma função $\omega(z)$,

$$\frac{\partial\omega}{\partial z}=\frac{1}{2}\left(\frac{\partial\omega}{\partial x}-i\frac{\partial\omega}{\partial y}\right);\quad \frac{\partial\omega}{\partial\bar{z}}=\frac{1}{2}\left(\frac{\partial\omega}{\partial x}+i\frac{\partial\omega}{\partial y}\right).$$

Elas são invocadas sempre que se deseja descrever localmente a estrutura de expressões complexas. Seja a Jacobiana

$$J=\left|\frac{\partial\omega(z)}{\partial z}\right|^2-\left|\frac{\partial\omega(z)}{\partial\bar{z}}\right|^2.$$

O comportamento local da função $\omega(z)$ é estabelecido pela equação de Beltrami,

$$\frac{\partial\omega(z)}{\partial\bar{z}}=\mu(z)\frac{\partial\omega(z)}{\partial z}, \qquad (5)$$

onde $\mu(z)$ é uma função complexa não-analítica sobre o plano complexo (não confundir com a magnificação μ), de maneira que para

$$J=\left|\frac{\partial\omega(z)}{\partial z}\right|^2-\left|\frac{\partial\omega(z)}{\partial\bar{z}}\right|^2=0,\quad |\mu(z)|=1.$$

Uma forma de visualizar aquele comportamento local de $\omega(z)$ é, de acordo com Schramm, por via do chamado "campo de elipses" do mapeamento extraído da equação de Beltrami. A Figura 2 mostra como um campo de elipses teórico e idealmente alinhado é distorcido por uma lente pontual (supondo simetria radial). Discuti com Schramm acerca de alguns aspectos do efeito de lenteamento e adicionei comandos ao código original por ele desenvolvido para exibição da curva crítica e construção da cáustica correspondente (Figura 3, $J = 0$) [14]. Para as galáxias elípticas (elipses vermelhas na figura), hipoteticamente dispostas em configuração horizontal, a lente causaria uma deformação como a que aparece na vizinhança da curva crítica (círculo vermelho). Nesse caso, a equação da lente no plano complexo é definida em código Maple pela função ω :=z→0.5*z-k/conjugate(z), onde k fixa a extensão da lente.

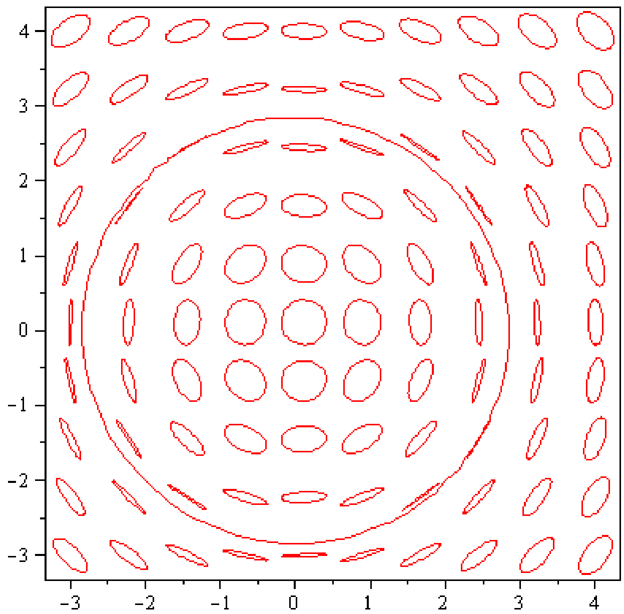

Figura 2 – Campo de elipses deformado pelo efeito de lenteamento para $k = 4$ (simetria radial perdida pela introdução do *shear*).

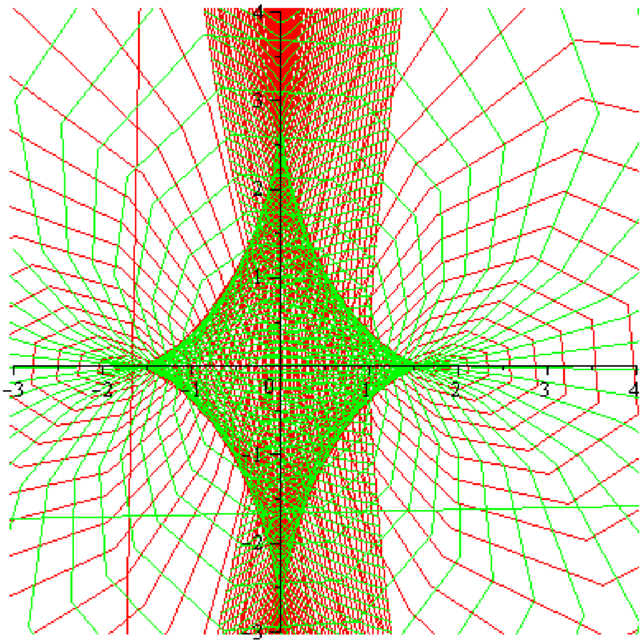

Figura 3 – Cáustica referente à curva crítica da Figura 3. A densidade de linhas é relacionada com a densidade de luz.

A Figura 4 ilustra o efeito de lenteamento, porém, considerando como fonte não um ponto massivo, mas uma galáxia elíptica. Nela o leitor poderá perceber com mais clareza o fenômeno da magnificação. Utilizei um código em MATLAB, criado por Newbury e Spiteri [9], e adaptado por mim para a geração de imagens segundo o modelo da esfera isotérmica singular (EIS), manipulando o parâmetro de massa e os *redshifts* da lente e da fonte. Cabe lembrar que nesse modelo o conteúdo massivo da lente é assumido ser um gás ideal constringido por um potencial gravitacional de simetria esférica. Tal gás é suposto em equilíbrio térmico e hidrostático. A fonte é representada pela pequena elipse magenta e o anel de Einstein pelo grande círculo vermelho. A magnificação diverge ao longo desse último de acordo com uma escala de cor; quanto maior a magnificação, mais nos dirigimos para o extremo vermelho do espectro.

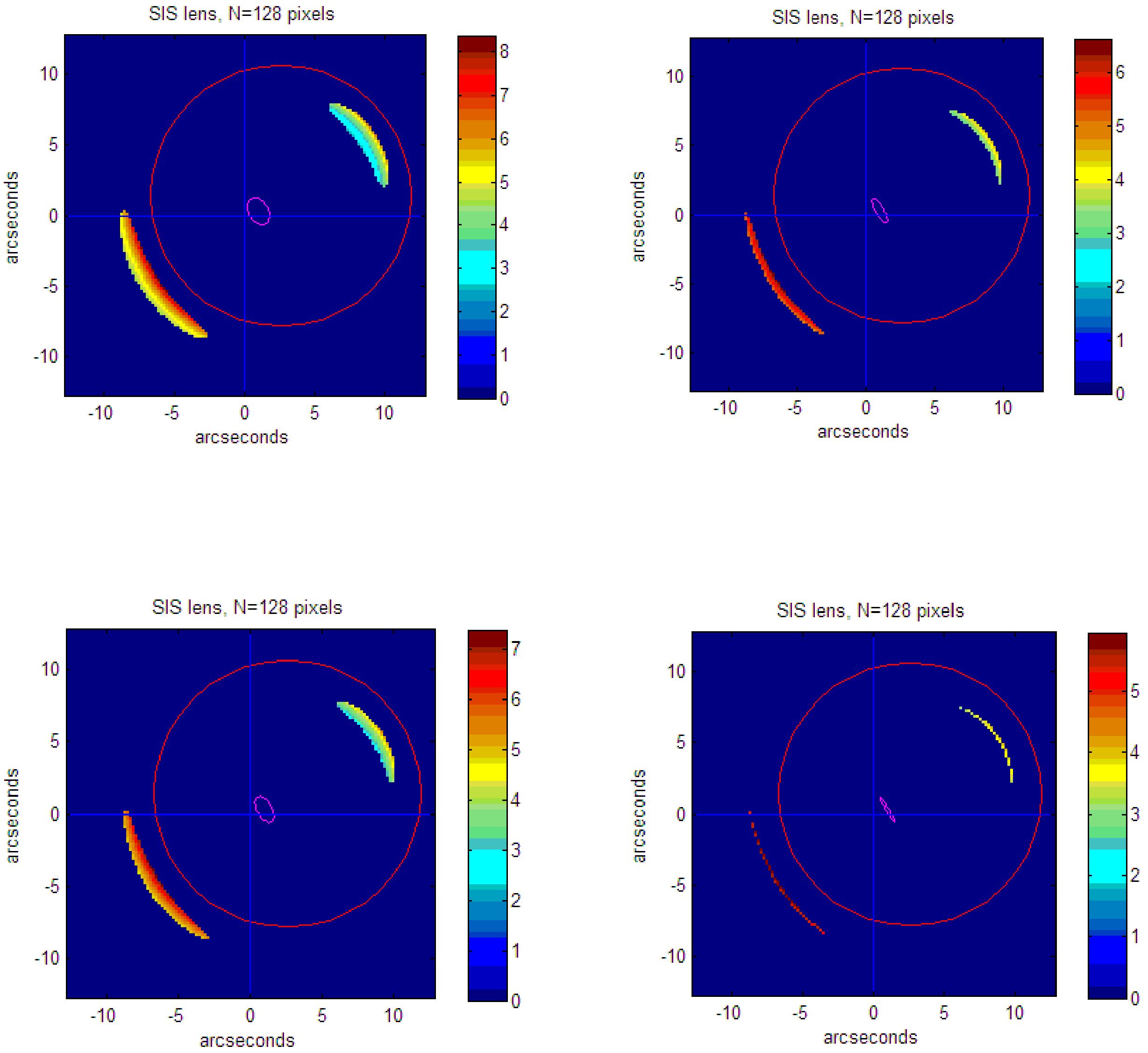

Figura 4 - De cima para baixo, da esquerda para direita: sequência de simulações para quatro elipticidades da galáxia fonte (0.3, 0.5, 0.7 e 0.9).

Os parâmetros da lente são as coordenadas do centro de distribuição de massa da EIS, o parâmetro de massa σ (= 750) e o redshift da lente (= 0.3). Os parâmetros da fonte são o redshift (= 0.7), as coordenadas do centro da fonte, a elipticidade e o fator de magnificação (= 1.0).

Na discussão acima eu considerei uma lente gravitacional a uma distância angular fixa de uma estrela. De fato , a separação angular está sempre mudando em função do movimento relativo da lente e da estrela. Tal fato implica em que o brilho total das duas imagens se altera de modo peculiar.

3. O Grupo Local

O Grupo Local é um pequeno grupo de cerca de 50 galáxias, a maioria delas anãs distribuídas ao redor da Via-Láctea e da M31 em amplitude radial máxima de aproximadamente 1.2 Mpc, supondo um potencial de simetria esférica. Em geral não se comenta o porquê da simetria esférica, mas podemos dizer que 1) ela serve para tornar a teoria e os cálculos mais simples, além de que 2) tudo que observamos está no passado, ao longo de uma linha radial que se aprofunda em qualquer direção desde um observador convenientemente posicionado no "centro".

A busca de novos integrantes do Grupo Local parece uma tarefa contínua. Muitos objetos foram descobertos em anos recentes. A dinâmica observada das galáxias ditas "satélites" do Grupo Local sugere a existência de regiões dominadas por halos escuros contendo 10 vezes mais matéria escura do que as satélites conhecidas. Do ponto de vista dinâmico, por menos que se admita, modelos de MOND (Modified Newtonian Dynamics, Milgron 1983) podem dar conta do cenário sem lançar mão de matéria escura, apenas alterando a segunda lei de Newton em baixas acelerações. Dessa forma, as objeções feitas às teorias de MOND deveriam discutir tão somente os limites de sua generalização a outros fenômenos. O fato é que MOND se torna mais complexa quanto mais complexo for o contexto estudado (por exemplo, é preciso introduzir o formalismo vetorial/tensorial para fazer cosmologia com MOND). Entretanto, até o presente momento, MOND não pode ser refutada como recurso de modelagem galáctica.

O grupo local é bastante heterogêneo em morfologia, idade, metalicidade e graus de isolamento. Galáxias anãs elípticas (dEs) e esféricas (dSphs) concentram-se em torno das grandes espirais; galáxias anãs irregulares (dIrrs) tendem a permanecer mais afastadas, muito embora as duas mais massivas (Grande Nuvem de Magalhães e Pequena Nuvem de Magalhães) estejam bem próximas à Via-Láctea e interajam com ela e entre si. Tal distribuição está longe de ser completamente entendida. Por suas características de baixa metalicidade e níveis relativamente altos de gás, as dIrrs têm sido consideradas similares às primeiras galáxias do universo.

As galáxias dIrrs apresentam traços de formação recente de estrelas. Várias delas possuem aglomerados globulares e abertos. Já as dSphs são dominadas por estrelas velhas ou de idade intermediária, sendo sistemas muito pobres em gas. A ausência de gás nesses objetos tem sido difícil de explicar, sobretudo porque há evidências de episódios recentes de formação estelar em algumas dSphs. As galáxias dEs têm concentração central bem pronunciada, ao contrário de suas vizinhas esferoidais com pequenos adensamentos em seus centros. Tanto galáxias dIrrs de baixa massa como várias galáxias dSphs exibem vestígios centrais de suas últimas criações estelares, fato que permite estabelecer um gradiente radial de idade associado a outro de metalicidade.

As galáxias espirais são sem dúvida os indivíduos mais complexos. Dotadas de estrelas de todos os tipos, idades, metalicidades e comportamentos cinemáticos, elas dominam gravitacionalmente o Grupo Local. Seus discos finos, berçários de novas estrelas, são preenchidos por nuvens moleculares e dominados por estrelas de população I. Nos discos espessos e nos halos predominam estrelas de população II, sendo que as estrelas extremamente pobres em metais dos halos são bem mais difíceis de datar.

Grosso modo, as estrelas são agrupadas em duas classes gerais ditas "População I" e "População II". A classificação depende de fatores como composição química, presença de gás nas redondezas, localização na galáxia, cor integrada e outros aspectos, mas, fundamentalmente, podemos afirmar que as estrelas da população I têm elevada metalicidade enquanto que as da população II apresentam baixa metalicidade. Tal diferença está diretamente relacionada com a idade das estrelas; as mais jovens, concentradas no disco fino, são ricas em metais, ao passo que as mais antigas, típicas das regiões externas e do halo, são pobres em metais. No cômputo geral, essa imagem atende à realidade observada, uma vez que a região do disco galáctico concentraria a matéria precipitada do halo durante o processo de formação da galáxia. A formação de nuvens moleculares no disco fino e o gradativo enriquecimento do meio interestelar pelos sucessivos ciclos de esvanecimento e geração de estrelas garantem a alta metalicidade do disco em detrimento das antigas e quase isoladas regiões do halo. Naturalmente, essa classificação é simplista e não resolve as subpopulações intermediárias, porém, serve para destacar a metalicidade como fator crucial no estudo da evolução estelar.

Como demos a entender acima, a heterogeneidade do Grupo Local nos leva a traçar histórias bem distintas para cada objeto. O entendimento da diversidade das galáxias anãs é algo que ainda nos escapa, apesar das modernas técnicas fotométricas empregadas na derivação das suas prováveis origens e da quantidade de dados acerca da cinemática relativa desses objetos e de suas dinâmicas internas. Contudo, elas compartilham padrões globais como metalicidade média, magnitude absoluta e brilho superficial central. Parece claro que suas massas e interações com o meio externo desempenham papel fundamental em cada estágio evolutivo. Por último, cabe lembrar que, apesar do Grupo Local ser relativamente pouco profundo e denso para produzir efeitos apreciáveis de lenteamento gravitacional, não se descarta a possibilidade de termos em futuro não muito distante alguns resultados de interesse, provenientes da varredura realizada por telescópios de grande resolução.

4. Lenteamento no Grupo Local

Como observou o astrofísico Bodan Paczynski em 1986, se o halo galáctico contém objetos escuros, chamados *Massive Compact Halo Objects* (MACHOS), com massas que variam desde a de Júpiter (~ 1/1000 da massa solar) até a das anãs marrons (~ 1/10 da massa solar), então tais objetos ao cruzarem a linha de visada entre a Terra e as estrelas mais distantes poderiam provocar efeitos de microlenteamento.

Quatro equipes de colaboradores — EROS, MACHO, OGLE e DUO — se empenharam na busca por objetos do halo, usando estrelas da Grande Nuvem de Magalhães (GNM) como referencial de *background* contra o qual se investigam os movimentos daqueles objetos [1].

A Figura 5 mostra um esquema de como ocorre o lenteamento no domínio em questão. A luz proveniente de uma estrela na GNM é defletida por um objeto massivo do halo. Tal objeto atua assim como uma lente gravitacional. Em tese, por meio da medição da distribuição de eventos de lenteamento se poderia inferir alguma informação acerca da natureza e da distribuição dos objetos do halo.

Na prática, procurar essas lentes é uma tarefa bastante ingrata, pois o tipo de evento descrito acima ocorre muito raramente. A colaboração EROS, por exemplo, é capaz de monitorar cerca de 4 milhões de estrelas ao mesmo tempo [1]; em três anos de varredura foram encontrados apenas dois eventos classificáveis como resultantes de lenteamento. Por outro lado, a equipe MACHO registrou vários candidatos ao fenômeno.

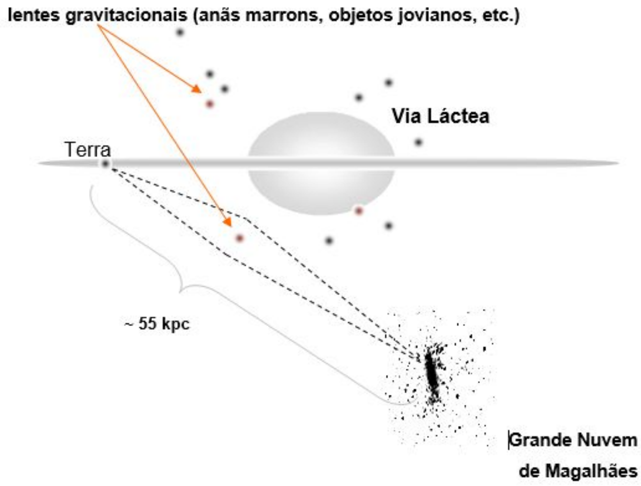

Figura 5 – Vista esquemática do sistema da Via-Láctea com ênfase na Grande Nuvem de Magalhães e em objetos massivos do halo capazes de produzir lenteamento.

Retomemos o sistema de lenteamento típico mostrado na Figura 1. Suponhamos que existam vários objetos lenteadores no campo de visão do observador. Definimos a profundidade óptica para o lenteamento gravitacional como o ângulo sólido que confina tais objetos e seus respectivos anéis de Einstein dentro do campo de visão.

Agora, suponhamos todos aqueles objetos com a mesma massa *M*. Assumindo a distância D_{lo}, sobre uma "lâmina" plana hipotética com espessura ΔD_{lo} podemos projetar em média uma lente por área superficial

$$\pi R_M^2 = \frac{M}{\rho \Delta D_{lo}}, \qquad (6)$$

onde ρ é a densidade de massa média, considerando todas as lentes no volume $\pi R_M^2 \Delta D_{lo}$. A secção reta de cada lente é dada por πR_E^2 (R_E é o raio do anel de Einstein). A contribuição da lâmina para a profundidade óptica é

$$\Delta\tau = \frac{\pi R_E^2}{\pi R_M^2} = \left[\frac{4\pi G\rho}{c^2}\frac{D_{lo}\left(D_{os}-D_{lo}\right)}{D_{os}}\right]\Delta D_{lo}.$$

Logo, a profundidade óptica total dentro do campo de visão do observador, devida a todas as lentes entre a fonte e o observador, é a integral

$$\tau = \int_0^{D_{os}} \frac{4\pi G\rho}{c^2}\frac{D_{lo}\left(D_{os}-D_{lo}\right)}{D_{os}} dD_{lo} \therefore$$

$$\tau = \frac{4\pi G}{c^2}\int_0^{D_{os}} \rho \frac{D_{lo}\left(D_{os}-D_{lo}\right)}{D_{os}} dD_{lo} \therefore$$

$$\tau = \frac{4\pi G}{c^2}\int_0^{D_{os}} \rho \frac{D_{lo}D_{os}\left(1-D_{lo}/D_{os}\right)}{D_{os}} dD_{lo} \therefore$$

$$\tau = \frac{4\pi G}{c^2}\int_0^1 \rho D_{lo}D_{os}\left(1-x\right)dx \;\; ;$$

$$(x = D_{lo}/D_{os} \therefore dx = \frac{1}{D_{os}}dD_{lo}) \therefore$$

$$\tau = \frac{4\pi G}{c^2}\int_0^1 \rho x D_{os}D_{os}\left(1-x\right)dx \therefore$$

$$\tau = \frac{4\pi G}{c^2}D_{os}^2\int_0^1 \rho x\left(1-x\right)dx \; .$$

Dessa forma, a profundidade óptica total depende da massa total das lentes, não das massas individuais M. Para densidade constante, temos que

$$\tau = \frac{4\pi G}{6c^2}\rho D_{os}^2 = \frac{2\pi}{3}\frac{G\rho}{c^2}D_{os}^2.$$

Se o sistema de lentes é auto-gravitante, supondo a distância D_{os} igual ao tamanho do sistema inteiro, o teorema do virial garante que

$$\frac{GM_{tot}}{D_{os}} \approx \frac{G\rho D_{os}^3}{D_{os}} \approx V^2, \qquad (7)$$

onde V^2 é a velocidade de dispersão e ρ a densidade dada por

$$\rho \approx \frac{3M_{tot}}{4\pi D_{os}^3} \approx \frac{3}{4\pi G}\left(\frac{V}{D_{os}}\right)^3.$$

Combinando os resultados acima, deduzimos que

$$\tau \approx \frac{V^2}{c^2}.$$

Para a GNM, $\tau \approx \frac{V^2}{c^2} = \frac{10^4}{9\times 10^{10}} = 0.11\times 10^{-6}$.

O monitoramento do espaço na direção da GNM analisou dados de milhões de estrelas ao longo de vários anos. De acordo com a colaboração MACHO (USA/Austrália), a profundidade óptica deduzida dos possíveis eventos de microlenteamento (*lasting* entre 2 e 400 dias) é

$$\tau_{LMC(MACHO)} = 1.2^{+0.4}_{-0.3}\times 10^{-7}.$$

resultado totalmente compatível com o nosso cálculo anterior aproximado e que implica no fato de que o halo da Via-Láctea pode ser formado por MACHOs numa taxa entre 8% e 50%, e numa faixa de massas entre 0.15 $M_\odot$ e 0.9 $M_\odot$.

5. Conclusão

O lenteamento gravitacional, sobretudo no regime fraco, é um instrumento útil para estabelecer parâmetros de modelagem do Universo e para compor a descrição da sua história de expansão e evolução. Permite efetuar cálculos acerca do modelo cosmológico atualmente mais aceito, tais como a densidade total da matéria. Também se mostra promissor para testar teorias ligadas ao modelo padrão, para restringir as leis da gravidade em escalas muito grandes ou para avaliar propriedades da energia escura e da matéria escura. Tenho expectativa de que essa revisão motive jovens estudantes para o aprofundamento do tema no Brasil.

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