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Cosmological Notes: Some Technical Insights to Warpdriving Cosmic Structure; Odd Galaxies; Life under Other Stars; General Comments

(In continuation to **Stylophilic Notes of a Physicist: Science and Philosophy Recorded by Fountain Pens**)

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Abstract - This essay is, in some sense, a continuation of the work entitled "*Stylophilic Notes of a Physicist: Science and Philosophy Recorded by Fountain Pens*", published in CALIBRE Vol. 9 (2024), but it is more technical about the physics of space-time and cosmology. Nevertheless, the references to the vintage fountain pens used in the preparation of the manuscript remain relevant in the spirit of the original proposal, which exalts the art of writing and doing theoretical science with old fountain pens, some of which may be centennial. The essay reflects a lifestyle and, above all, a peculiar way of producing and recording knowledge, at the same time exciting and stimulating, considering the reiterated benefits of handwriting for the brain, and the need to protect it from the binary adaptation induced by generative AI, one of the biggest hoaxes of all time. It is expected that the essay will motivate readers to practice handwriting, especially with fountain pens, and to study astrophysical cosmology with emphasis on the structure of the space-time woof.

Keywords: Fountain pens, Space-time, Gravity, Cosmology, Inhomogeneous universe, Reentrant galaxies, Metamaterials.

There is nothing worse for science than the figure of authority. An authority, in the moving scenario of scientific activity, would be a contradiction, unless one accepts an authority in collecting all knowledge that humanity does not understand.

1 Introduction

It is impossible to separate the development of modern physics from fountain pens, at least during the peak of great discoveries until the 1940s. Einstein, De Broglie, Heisenberg, Somerfeld, Gábor, Schrödinger, Bohr and many others certainly wrote their theories with excellent nibs (regardless of the price of pens); beautiful works of reason, written by amazing minds, which grounded the technology that younger generations boast of with their flashing trinkets, as if they had created them from nothing or from fictitious strokes of genius. Here is one of the great fallacies of the 21st century: comparing, on an intellectual level, Silicon Valley with the scientific and philosophical schools of the early 20th century, such as Göttingen, Copenhagen, Princeton, Vienna and many others. The latter symbolize science and philosophy in its great and interconnected intellectual process; the first is simply, and at best, the plutocratic symbol of the replacement of running science by successive technological collages, whose subjacent theoretical knowledge, which some technocrats joke they are its co-creators, dates back to the first decades of the twentieth century. This is not about detraction or disputes of intelligence. They are simply two examples of human social activity that do not touch on each other in their origins, much less in their objectives. This comparison, which I have heard many times, is part of the superficiality of postmodernist thinking, and ends up bringing undesirable idiosyncrasies to education. The postmodern stance certainly accelerated the decline of interest in formal education, philosophy, literature and writing, culture in general, and, consequently, vintage fountain pens, giving priority to chatbots and the monosyllabic language of cell phones.

Talking about interesting stuff, the evolution of materials technology is easily noticeable if we follow the evolution of fountain pens since the beginning of the 20th century. Among the first popular materials, vulcanite, ebonite or vulcanized rubber, appears to have been the most widespread in black, the much-demanded color. This material often formed the basis for applications of gold filigree, as well as for elaborate finishes in chasing elevated patterns. These applications came to coexist with the use of other materials that were being improved, such as lucite (polymethylmethacrylate), galalite (casein) and the laminated celluloid popular among Parker vacumatics. All these historical curiosities, unfortunately, have no appeal to practically anyone in present days. At the same time, it seems that most people are not concerned with good writing and real science, turning eyes almost exclusively to everyday technologies.

There is, of course, a new generation of pen fans, however, just wanting modern pens more as symbols of the contemporary world accompanying their expensive wristwatches. Ok, no problem with that because there are many excellent new pens out there. There is also, as I have already discussed, the issue of a fair relationship between cost and benefit, taking into account the ridiculously high prices charged by some sellers of old pens. Nevertheless, I think that the main driver of the decline in demand for old fountain pens, as I have been saying, is

the abandonment of mastering writing and quality reading. Therefore, anything that writes common things will do. The nib variety of those older pens has become irrelevant. Writing and reading are no longer pleasures for most.

Figures (1) - The Conklin Toledo with patent imprints, 14K gold-filled, lever fill.

Figure 2 - The Onoto clipless, 1915, 14K solid fine gold nib and bands.

Figures (3) - The Aurora 88K, 1950, 14K gold-filled cap, with piston filler.

Figures (4) - The British Esterbrook Relief, manufactured by Conway-Stewart, 14K gold nib, lever fill.

Fountain pens wrote the history of the two great theories of physics of the 20th century, as well as some of the famous notes and letters among the scientists who proposed them. There were different times. Reading books taken off the shelves and writing by hand were natural activities that stimulated mental connections. Fountain pens were lovely instruments for their owners and accompanied them in the feverish moments of creation. They were also a subject of propaganda among famous actors (Figure 4a).

Figure 4a - The famous actress Ava Gardner writing a letter with a fountain pen (credits to Clarence Sinclair Bull / Getty Images).

It is true that the development of new communication technologies (pens are also communication technologies!) have brought independence, speed, and convenience. We now have almost instantaneous access to the world's scientific productions, and can easily correspond with their authors. All of this is very positive, no one doubts it. What is not at all positive is to accumulate knowledge without understanding (which, by the way, is what is most often done today!). What is also not positive is making that modernity an aim in itself to the detriment of the elementary processes that constitute an irreplaceable part of the functioning of the creative and associative mind. I refer to the brain-hand-writing relationship and the brain-hand-book-reading relationship. Picking up books with different colors, textures and smells stimulates associations with olfactory and tactile

memories, creating records of motivations and useful references to the mental attitude aimed at discovery. If we lose these connections, it will be difficult for us to recognize ourselves as human beings in the future.

2 The handwritten cosmology: A new look recorded with old fountain pens

I recently added several items to my collection of old fountain pens. Some of them are a bright Conklin Toledo (Figures (1)), an Onoto self-filling — the best of the best — (Figure 2), a magnificent 1950s Aurora 88K (Figures (3)) that I found on the Internet (in fact, the only Aurora that really caught my attention), and a delicious English Esterbrook Relief by Conway-Stewart (Figures (4)). They covered much of the text of present essay. The Conklin brand has always aroused my curiosity due to the association of its crescent filler model with the writer Mark Twain, who I greatly admire[1]. This is not my preferred design, but Conklin has lovely models, one of them, my favorite, a Toledo under patent 5-28-18, gold filled, with 14K gold nib, showed above; I am already enjoying using it to write about cosmology and cosmological time.

A parenthesis here. Time, as I said elsewhere, has always been my passion in physics, and so it pleases me to think that the reading of the fictional literature about time travel has somehow influenced this feeling. Nonetheless, the story that has had the greatest impact on my meditations on time is on the television screens of Star Trek Voyager, in the episodes "Year of Hell", Parts I and II. Annorax (in the shoes of veteran Kurtwood Smith), a brilliant alien temporal scientist of the Krenim Imperium, was an officer self-imbued with the obstinate mission of restoring the power and grandeur of his people. In possession of a sophisticated temporal technology consolidated in a fantastic spaceship, he triggers a series of disruptions in the structure of space-time, but the countless variables involved in the temporal incursions he deployed never conspired in favor of the restoration of his homeworld, Kyana Prime, nor the power of the Krenim Imperium. In your agonizing saga, isolated from cosmological time with his crew within the exotic temporal vessel, he struggled over centuries forcing the time in vain to return his wife to him, while witnessing the repeated disappearance and reconstruction of countless civilizations. A fantastic story! I think I identify a little with Annorax in the sense that in some way I try to restore the glory of writing as it was done in the past; a solitary and echoless attempt.

Continuing about the recent acquisitions, the Onoto self-filling, fine nib and two rings, all made in solid gold, by Thomas De La Rue Ltd, clipless in hard rubber, has an advanced technology considering the time it was launched, around 1914. Its self-filling system was later implemented by Sheaffer. In my opinion, the Onoto is superior to all their contemporaries, including Watermans. Even though it is not a brand of special interest for me,

[1] Twain was fascinated by technology and scientific research, having even befriended Nikola Tesla. His book "A Connecticut Yankee in King Arthur's Court" [1] inspired later time travel stories I read.

I recognize that the elegant and stylish Aurora 88K — perhaps the most famous vintage model from the Italian brand Aurora — is one of the best-balanced fountain pens I have. Lastly, the Esterbrook Relief in hard rubber by Conway-Stewart, nice gold filled medium nib, with the customary elegance of the British brands.

We live in an inhomogeneous universe; that has always been my belief, although the isotropy of the cosmic microwave background may be part of an argument to the contrary (for me, such isotropy serves mainly to corroborate cosmic time). The general context of the research I conducted during many years on inhomogeneous models of the universe — exact solutions of Einstein's equations in which quantities such as curvature and expansion field vary point-to-point in space-time — was based on the classical cosmology of Friedmann, Lemaître, Robertson, Walker (FLRW), from which inhomogeneity was introduced. All theoretical and interpretative efforts undertaken by most cosmologists characterized an obstinacy to preserve a homogeneous description, in my opinion quite controversial, and has given rise to a range of works aimed at increasingly affirming the inhomogeneous image that technological modernity, in obtaining precise data, has allowed us to glimpse, especially with regard to the study of type-Ia supernovae (SNIa), the most breathtaking stellar explosions in nature. At the time I started the first work, my main task was to describe a simple inhomogeneous Lemaître-Tolman (LT) universe viable with the same level of detail achieved in the description of its homogeneous alter ego FLRW.

My cosmology bases on four elementary premises, each one written here with a different pen:

1. The universe is inhomogeneous → Conklin Toledo;
2. Space-time is a continuously expanding four-dimensional manifold → Onoto self-filling;
3. On Planckian and sub-Planckian scales, space and time can configure, under special conditions, inversions of symmetry (three dimensions of time and one of space) → Aurora 88K;
4. Gravity hides predominantly in a fifth dimension embedded in the fourth dimension → Esterbrook Relief.

The first premise is still polemic. The hypothesis of a homogeneous universe can be interesting; certainly, it simplifies the theoretical foundation and brings promising results, but there is an ontological problem in the intimate relationship between man and the universe: we are inside the universe and cannot encompass it completely. Furthermore, inhomogeneities occur in everything we reach with our instruments. A persuasive example is the so-called *KBC* void, referring to the large vicinity of the Milky Way, whose average density of matter is notably lower than the average in the observable universe. The photometric characteristics of SNIa themselves seem to depend on the environmental conditions of their birthplaces, whether their host galaxies are found in clusters or in open fields. In addition, as I reported in previous work [2], an accurate work by Migkas and colleagues [3] discusses the common assumption of isotropy of the late Universe, testing the anisotropy on the X-ray galaxy cluster scaling relations with important outcomes following many studies reporting deviations from isotropy under different cosmological probes. Importantly, galaxies lose the raw material (cold gases) for making new stars when they are dragged into giant clusters, thus reducing the local rate of star generation. Yet, it is also evident that life is a rare phenomenon, so that biological activities constitute a largely inhomogeneous scenario. Therefore, a homogeneous universe is commonly defended, but inhomogeneities bring us understanding about the most impressive nuances of its evolution.

The second premise conflicts with the confusing ideas on quantizing general relativity. Thinking in terms of laboratory apparatuses, we may say the starting problem is that gravity is extraordinarily weak and, although it seems to act as a force in common sense, it is much more than this, since gravitational field is the deformation of the shape of space-time itself. Furthermore, gravity is manifested by effects that are the result of a long cumulative process over time. While quantum theories zoomed-in on something as small as neutrinos and quarks, and may

treat space as a flat background for measuring how far particles interact, putting time as an external counter and ignoring the curvature of space-time, the effects of gravity only become evident at the very zoomed-out levels of massive bodies like planets, stars, black-holes and so on, objects resulting from an age-old cosmic evolution. At this scale, time demonstrates its physical reality as an evolutionary variable, playing a clear creative role. All of these features characterize widely divergent contexts. In consequence, an enormous amount of failed attempts to quantize gravity have accumulated since the 1930s in view of serious theoretical and experimental drawbacks that were arising and adding to, obviously because 1) there is no concrete empirical evidence of the quantum emissary of the gravitational interaction (graviton), even less of its supersymmetric partner (gravitino), and 2) the formalism of quantum field theory is intended to describe specific empirical contexts by which it is justified; apropos, renormalization stops running when the hypothetical gravitons enter the game, generating a great math-mess from an endless feedback-loop of *space-time warping - creation of gravitons - space-time warping again*. To apply quantum field theory under such circular conditions would be forcing gravity to behave in a way that fits an empirical context completely outside of its reality. Even loop quantum gravity, whilst interesting and theoretically innovative, resent a hard lack of testable predictions. At least for now, I think we waste time trying to paint gravity with conventional quantum hues, hunting gravitons and gravitinos perhaps as exotic elements very difficult to detect (instead, we can work on a supersymmetric semiclassical theory of gravitation via adS space-time glued to Minkowskian space-time).

The third premise reports to non-locality. The relationship between space and time that we experience is not unique. In a quaternionic representation, a Wick-rotation of Minkowski space-time leads to a different space-time, where "particles" interact freely through time (in three dimensions of time: past, present and future), however, limited to a single spatial dimension. My theory starts from two basic questions: is there a symmetry (a perfect symmetry) between space and time that makes possible information traffic from present to future as well as to past? Moreover, if so, is there a phenomenon that insinuates this possibility? According to my interpretation, a perfect symmetry between space and time determines that both can be converted into each other. In other words, the reverse of space is time, and vice versa. To answer the second question, the only phenomenon we have at hand to help us to justify such a transformation is the quantum entanglement. However, if space-time can suffer that presumed role reversal, it is reasonable to assume that this reversal begins with quantum entanglement itself, since

entangled particles, separated by an arbitrary distance, change their behavior simultaneously as if they broadcast freely between past and future. The role reversal, I believe, occurs at the precise moment of quantum entanglement. As space-time continually expands, past and future are as mobile as the present. Thus, the present is an important coordinate because it is what anchors the entanglement in the flow of time.

Lastly, the fourth premise accounts for the weakness of gravity (it is four-dimensionally weak because it leaves mainly in the embedded fifth dimension), and is closely related to the previous premise. Of course, if we think of space (or time) reduced to a "filament" — a one-dimensional slice of the cosmic woof — it will be natural to deduce there may be an infinity of connected filaments. Assuming conventionally a fifth dimension that answers by such a connection would lead us back to the old question of why we do not perceive it. In addition, we would add an unnecessary math complication.

In general, when reference is made to an extra dimension in cosmology, the abstraction is more a mathematical exercise on the expression of the geodesic arc's quadratic element than a physically motivated description provided by an ontological intuition about the observed phenomena. There is nothing new here concerning the ultimate essence of the continuum. The problem is that, in this context, the introduction of a fifth dimension is a conventional, adventitious and arbitrary geometric intervention, *ut regulae* the implementation of an angular dimension. With regard to inhomogeneous models, the effectiveness of applying metrics with more than four dimensions is still an open issue, mainly because, as far as I know, the consideration of such metrics is practically restricted to discussions about the emergence of naked singularities. There is still no consensus on the realism of such objects from the point of view of general relativity, and there is little evidence of the productive applicability of metrics with more than four dimensions connected to cosmology observational data.

Briefly, from a geometric point of view, the implementation of a fifth dimension is quite simple if we consider the symmetries involved. The symmetries of a space-time, or isometries, constitute a group for which a) the identity is an isometry, b) the inverse of an isometry is an isometry, and c) the composition of two isometries is an isometry. The orbit of a point p is defined as the set of all points to which p can be moved by the translational action of the isometries of the space. The orbits are necessarily homogeneous, that is, all physical quantities are the same at each point. Since an invariant manifold is a set of points mappable onto themselves by the group of

isometries, the orbits are necessarily invariant manifolds. The freedom of translation in a given space, or translation dimension, is generally symbolized by the letter "s", and the expression $s \leq n$ is defined, where n is the number of dimensions of space. An important subgroup of the group of isometries, whose dimension can be considered at each p, is the group of isotropies, that is, the group of isometries that leave p fixed (rotations). In general, the rotation dimension of a space is represented by the letter "q", defined by the expression $q \leq 1/ [2n(n-1)]$, where n is the number of dimensions of the space. Thus, the dimension $\mathfrak{D}$ of the group of isometries of a given space is $\mathfrak{D} = s + q$ (translations + rotations). In fact, continuous isometries are generated by the Lie algebra of Killing vectors. The action of the group is characterized by the nature of the respective orbit in the space in question. For a cosmological model, due to the four-dimensionality of space-time, the possible orbital dimensionalities are $s = 0; 1; 2; 3; 4$.

The isometric group that characterizes *4D* LT models is $\mathcal{G}_{S+q} = \mathcal{G}_3$ or $\mathcal{G}(2;1)$, isomorphic to the special pseudo-orthogonal real group on $s + q$, **SO**(*2;1*). Each LT model is characterized by a two-dimensional surface of spherical symmetry: $s = 2$; all observations made at any point on the surface are rotationally symmetric about a privileged spatial direction: $q = 1$; therefore, $\mathfrak{D}_{4D\ \mathrm{LT}} = 2+1 = 3$. However, the implementation of a fifth angular dimension corresponds to the introduction of an extra "translational" degree of freedom, $p = 1$, whence $\mathfrak{D}_{5D\ \mathrm{LT}} = s + p + q = 2 + 1 + 1 = 4$. Consequently, a *5D* LT model requires an isometry group $\mathcal{G}_4$, isomorphic to the orthogonal singular group in $s + p + q$, **SO**(*2; 1; 1*), corresponding to the inhomogeneous Lie algebra **so**(*2; 1; 1*).

The way described above for implementing a fifth dimension boils down to an exercise in geometry and group theory, which in my opinion is of little practical use for establishing a clear understanding of the essence of the space-time-gravity relationship. The representation of such a relationship must be supported by a structuring algebra that is, above all, predicative in the specification of its elements. Therefore, only a fifth-dimensional representation of space and time signatures, independent of cosmology, could be considered as a framework of structures through which gravity manifests itself. Better said, that framework makes gravity, making itself by gravity. An extra dimension is only justified if it defines a structural physical signature, enclosing a degree of freedom claimed by the phenomenon itself in such a way that it is as it is. Thus, the weakness of gravity compared to other interactions (fact) is just what nature lets slip from a hidden fifth dimension, embedded in the fourth dimension (be it time or space-like) through which the gravitational interaction navigates freely.

It seems elegant to suppose a hidden component embedded in the fourth dimension and signed by an imaginary unit distinct from the others, defined as $\varepsilon = \sqrt[4]{-1}$. We do not perceive it because it is part of the fourth dimension

(not because it is coiled upon itself, but because it is topologically “coiled” along the fourth dimension)[2], and because it is, by definition, null under Wick-rotation. In fact, the component associated with the fourth dimension, that is, the "bridge" between the infinite filaments of time or space, expresses what really constitutes the edifice of the continuum: four-dimensional slices that stand out from a "fifth direction" which is neither time nor space. In addition, more precisely, the qualitative difference between space and time, which perception imposes on us, does not belong to the objective world, as Weil has pointed out. Concisely, that is the nature of gravity: very weak in individual slices, but strong in a continuous woof of slices connected by the hidden fifth dimension, the “bridge” in between (see page 18). There would be no way to present the entire theory here, so the interested reader should access the reference [4].

Anyway, there are still those who insist on the idealist thesis that time does not exist, which for me, given the above, means that space does not exist either. I believe we can say that psychological time is just a mental construction based on subjective connections with the world of external things, therefore not having physical reality. Physical time is an evolutionary variable in synchronicity with entropy, that is, both go in the same direction, which does not mean there is a causal relationship between one and the other.

The entire theory I have summarized in the last few paragraphs clearly indicates that it is not my goal to discuss a unifying theory of the two great and successful views of the world. For me, the discrete images of reality are only constructs elaborated by the mind to interpret physical interactions captured by technology. Ultimately, space-time made up matter. Thus, the most logical way to conceive physical reality is to imagine a dynamic continuum formed by very small expansions (or contractions in some circumstances). Perhaps the best elocution to say it is "indefinitely small" expansions, but finite in some strange way. It is extremely helpful to analyze the distinction between the infinitely small of mathematics and the indefinitely small of physics. Lazare Carnot, in his classic "*Reflexions sur la Métaphysique du Calcul Infinitésimal*", made a magnificent explanation that serves as a conceptual-philosophical basis for the referred distinction:

"*En décomposant, pour ainsi dire, les corps jusque dans leurs éléments, elle* [*l’analyse infinitésimale*] *semble en avoir indiqué la structure intérieure et l'organisation; mais comme tout ce qui est extrême échappe aux sens et à l’imagi-nation, On n’a jamais pu se former qu'une idée imparfaite de ces éléments, espèces d'êtres singuliers, qui tantôt joueñt le rôle de véritables quantités, tantôt doivent être traités comme absolument nuls, et semblent, par leurs propriétés équivoques, tenir le milieu entre la grandeur et le zéro, entre l’existence et le néant.* " [5].

[2] It is possible to imagine that, under the action of extremely massive bodies, the fifth dimension "warped" over the fourth unfolds, enabling new forms of interaction. This could help us understand the physics of black holes. For now, it is just a conjecture as an educated guess, but we know that unfolding a dimension is something that depends on incalculable amounts of energy. Only bodies such as black holes and magnetars can offer environments capable of forcing dimensional unfolding.

From this, we conclude that for mathematics, at the infinitely small domain, existence and non-existence define the extreme limits of an effort of reason to understand what lies between them; for physics, it is inherent to the very nature of the indefinitely small expanding element of space-time to maintain itself in permanent transition between such limits, between being and nonbeing (it doesn't sound so strange to anyone who is familiar with quantum physics!).

The space-time dynamic continuum “cell”, expressed by that indefinitely small quantity, is the quantum of space-time[3]; no matter how small one can imagine, it can be still smaller and again smaller — remaining finite between existence and non-existence —, not because it is inaccessible to the rules, but because its dynamics runs in the domain of the indefinitely small steps. In this approach, what we call "dark energy" is the energy dissipated by indefinitely small fluxions of space-time. This understanding is ontological, without any inconsistency with current and successful theories in their fields of coverage.

I took a break from my Esterbrook Relief — no doubt, one of the best pens I've tried — to use a recently acquired 1938 Melbi from Merz & Krell (subsidiary of Merz Pharma), which is just as special, oversized and chased with gold filled nib, piston filling original mechanism, measuring closed 14 cm (Figures (5)). The first one had run out of ink, and I needed to complete those complex reflections without interruption, or I would lose the clarity of the moment. It is amazing how these pens inspire the intellect. Even the smell of old things, in combination with the shelves full of classic works, contributes to generating a nostalgic environment, conducive to reflection and far from the disgusting Brazilian politics that have little interest in education and science. In fact, in Brazil there is a political project to deconstruct education, which serves to predatory neoliberal interests, never minding the partisan orientation in charge. It is a vicious circle that never ends; it seems that even a well-intentioned person gets lost when dragged by political life. By the way, I am with Mark Twain: Politicians and diapers have a lot in common; both need to change frequently and for the same reasons.

[3] In my first approaches, I used the expression "quantum of space-time" as a way of keeping communication familiar with known vocabulary. Little by little, I have been restricting its use as much as possible as I deepen the ideas and publish the most relevant essays. In fact, the tiny virtual "cell" of the continuum is finite in thought, but indefinitely small in its sub-Planckian reality.

The idea of an indefinitely small but still finite object seems to contradict the notion of a continuum. Mathematically this would be true. However, in physics, indefinitely small means that we can take any tiny piece of space-time with the certainty that if it were smaller we would see the same dynamic behavior. Thus, thinking of a geodesic interval as indefinitely small, we want to mean that, from a physical point of view, whatever the scale taken, we will always have the same representation of the whole, that is, of the worldline. In other words, space-time just exists, and, as an objective form in itself, it remains the same, no matter how large is the interval imagined. This is fundamental to understand the evolution of the cosmic woof in terms of the structures that are being formed as the universe expands.

To present the modeling related to the discussion that proceeds, we will take Pauli matrices to set Wick-rotations,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

and the non-commutative products between imaginary unities,

$$\begin{aligned} ij &= k;\ ji = -k; \\ jk &= i; kj = -i; \\ ki &= j; ik = -j. \end{aligned}$$

The following representation in *2×2* matrix components was suggested in light of that any quaternion accepts a representation in *2×2* complex matrix form, such as

$$x = x_0 + x_1 i + x_2 j + x_3 k = z + qj \therefore$$

$$\begin{bmatrix} z & q \\ -\bar{q} & \bar{z} \end{bmatrix} = \begin{bmatrix} x_0 + x_1 i & x_2 + x_3 i \\ -x_2 + x_3 i & x_0 - x_1 i \end{bmatrix}.$$

Thus, there are matrices of Wick-rotating matrices, with Pauli matrices collected into quaternion basis for convenience. With all this in hand, we can Wick-rotate a quaternion of three temporal and one spatial coordinates into a Minkowskian quaternion of three spatial and one temporal coordinates by applying a matrix product, say,

$$\begin{pmatrix} O & \begin{vmatrix} 0 & +k \\ -k & 0 \end{vmatrix} & O & O \\ O & O & \begin{vmatrix} +j & 0 \\ 0 & +j \end{vmatrix} & O \\ O & O & O & \begin{vmatrix} +k & 0 \\ 0 & +k \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & O & O & O \end{pmatrix} \begin{pmatrix} +1_2 \\ +i\sigma_{(k)2} \\ +j\sigma_1 \\ +k\sigma_1 \end{pmatrix} = \begin{pmatrix} +\mathbf{i}_2 \\ -\sigma_1 \\ -\sigma_1 \\ -\sigma_1 \end{pmatrix},$$

where the *2×2* matrix components in the *4×4* matrix include quaternionic imaginary unities. Matrices of this type are in fact quaternionic basis transformation operators. The second component of the basis to Wick-rotate, $+i\sigma_{(k)2}$, refers to the Pauli matrix σ_2 but with *k* as the imaginary unit. There is a diversity of possible combinations, but only matrices that multiplied by their transposed conjugates, giving entries $\pm 1_2$, elegantly represent basis transformations.

As previously stated, the one-dimensional reduction of space or time, as constituting filaments, leads us to suppose a hidden physical connection intertwining them as part of the fourth dimension (the "fifth direction" that is neither time nor space represented by an imaginary unit $\varepsilon = \sqrt[4]{-1}$); the relative smallness of gravity compared to other interactions is probably because it largely inhabits this hidden dimension. In fact, the so-called "bridge" is nothing more than a construct — *une façon de dire* — that represents the gravitational adhesion between the filaments of space-time, which opposes its expansion.

To include the bridge in the model, we must define the following matrices:

$$\mathrm{E}_1 = \begin{bmatrix} \sqrt[4]{-1} & 0 \\ 0 & \sqrt[4]{-1} \end{bmatrix} = \begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix}; \mathrm{E}_2 = \begin{bmatrix} 0 & -\sqrt[4]{-1} \\ -\sqrt[4]{-1} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\varepsilon \\ -\varepsilon & 0 \end{bmatrix}; \mathrm{E}_3 = \begin{bmatrix} 0 & -\sqrt[4]{-1} \\ \sqrt[4]{-1} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\varepsilon \\ \varepsilon & 0 \end{bmatrix};$$

$$\mathrm{E}_4 = \begin{bmatrix} \sqrt[4]{-1} & 0 \\ 0 & -\sqrt[4]{-1} \end{bmatrix} = \begin{bmatrix} \varepsilon & 0 \\ 0 & -\varepsilon \end{bmatrix}.$$

That fifth direction, by definition, is null under Wick-rotations and "sluepotent" ($\varepsilon\varepsilon = i$). Then,

$$i\varepsilon(\varepsilon i) = j\varepsilon(\varepsilon j) = k\varepsilon(\varepsilon k) = 0; \varepsilon\varepsilon = i.$$

Lastly, to conclude these preliminaries, the bridge allows defining the superunitary matrix,

$$\begin{pmatrix} 0 & 0 & -\sigma_3 & 0 \\ 0 & 0 & 0 & -i\sigma_3 \\ -\sigma_3 & 0 & 0 & 0 \\ 0 & (i\sigma_3, E_4) & 0 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & 0 & -\sigma_3 & 0 \\ 0 & 0 & 0 & (-i\sigma_3, \pm E_4) \\ -\sigma_3 & 0 & 0 & 0 \\ 0 & i\sigma_3 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \tilde{U} \end{pmatrix}, \tilde{U} = 1 \pm i.$$

Right now, I am going to use a mix of nibs before refilling all the other pens in use. I will continue adding one more Melbi (12 cm) in a very nice black and pearl resin with Melbi 14 Kt gold nib, no iridium, button fill, from 1935 (Figures (5)), giving them the opportunity to write the mathematics that will follow.

Figures (5) - The nice Melbi duo.

As a physicist, I like and search symmetries. A symmetry between space and time as I imagine it makes the universe much more elegant. Furthermore, space-time is always generating space-time through its continuous expansion. Ergo, there are no real grains of space-time, only the continuum, perennially stretching out from the sizeless size. Finally, gravity hides in the fourth dimension with weak effects on the others.

Therefore, quaternionic representation as explained above is not limited to providing support for the understanding of quantum entanglement, since it is a formalized structure of the space-time continuum, which constitutes all things. We can construct transformation matrices with different physical contents and apply them on a specific quaternionic basis. We can even think of a flow of space-time with its own expansion in mind. The fundamental idea is that to act in four dimensions we need to conform the physics of external clocks within a framework consistent to the advocated complete symmetry between space and time. So, we can think of a practical application of this theory to develop in future times. Little by little, metamaterials technology has advanced, opening up perspectives for the construction of physical systems that we could call “exotic” compared to currently

known materials science. Metamaterials are materials with emergent physical properties not shared with common matter. We can conjecture about the discovery of a metamaterial that in the future can drive an accelerated flow of space-time, so that a piece of this metamaterial changes position not by its displacement, but by the displacement of space-time itself. Such a discovery would represent a true revolution in matter transport.

Let us imagine a rod made of an extremely dense metamaterial, subjected at one end to a powerful electromagnetic field in a configuration capable of producing an artificial gravitational field of great magnitude. In theory, the creation of this artificial field would be possible as a technological development of Füzfa's initial theory built simply on the equivalence principle [9], without any quaint physics. The hypothetical metamaterial would have the singular property of curving space-time in a dynamic folding, a "flow" of space-time along the rod's largest dimension, its length. This flow is established through the "conduit" of metamaterial by a density gradient along the rod, such that at its extreme end space-time unfolds abruptly, creating a strong impulse. The folding of space-time is intensified at the end of the rod chosen as the point of application of the artificial gravitational field. The density gradient is gentle to warrant the flow, that is, to maintain the folding along the rod. In practice, this model is simply the potentialization of the expansive nature of space-time itself, making it expand in the appropriate direction and geometric limits and with the appropriate intensity to propel the rod to truly prodigious speeds.

The entire algebra of signatures presented creates a complex skeleton on which physical interactions can be represented, including the movement of space-time through a given metamaterial surface (or metasurface, that is, metamaterial of reduced dimensionality [8]). Let, for instance, a transformation matrix represented by $\mathcal{T}_{4\times4}$ with sixteen 2×2 matrix entries, diffusing some physical quantities into the four-dimensional continuum through a $\mathcal{B}_4$ quaternion basis with 2×2 matrix entries, forming a quaternion field that crosses a parameterized metamaterial surface, say, $\mathcal{S}$, belonging to a manifold $\mathcal{M}$, $\mathcal{S}\in\mathcal{M}:\varphi(u,v)=\left(u,v,1-u^2-v^2\right)$, $(u,v)\in\mathcal{S}:0\le u\le1$ and $0\le v\le1$, such that, in a small region $D_{\mathcal{S}}:\left\{\left(\partial_u\varphi\times\partial_v\varphi,0\right)=\mathcal{G}\left(\varphi_u,\varphi_v\right)\right\}$ the field flux (induced) is defined as

$$\mathcal{F}(u,v)=\iint_{\mathcal{S}}\left(\mathcal{T}_{4\times4}(u,v)\times\ \mathcal{B}_4\right)\bullet\mathcal{G}\left(\varphi_u,\varphi_v\right)dudv\,.$$

Additionally, let us suppose a basis of spatio-temporal relations with three time signatures and one space signature[4]. Under quaternionic Wick-rotation, that is, under the transformation matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & k \end{bmatrix}, \qquad (4)$$

the small region $\varphi_u \times \varphi_v$ becomes a pseudo-Euclidean space-time tetrad with zero spatial component (which featured, for instance, de null distance between the entangled photons). So, as an example, let us take the transformation matrix

$$\mathcal{T} = \begin{pmatrix} O & \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & O & O \\ O & O & \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix} & O \\ O & O & O & \begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix} \\ \begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix} & O & O & O \end{pmatrix}. \qquad (5)$$

Now, let us take the basis

$$\mathcal{B} = \begin{pmatrix} +\mathbf{i}_2 \\ -\sigma_1 \\ -\sigma_3 \\ -\sigma_1 \end{pmatrix}. \qquad (6)$$

Transforming $\mathcal{B}$ with $\mathcal{T}$, we get a new basis

[4] Typical of the connection between two entangled photons. At their origin, the two photons virtually occupy the same position in the $\varphi_u \times \varphi_v$ small region.

$$\begin{pmatrix} O & \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & O & O \\ O & O & \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix} & O \\ O & O & O & \begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix} \\ \begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix} & O & O & O \end{pmatrix} \times \begin{pmatrix} +\mathbf{i}_2 \\ -\sigma_1 \\ -\sigma_3 \\ -\sigma_1 \end{pmatrix} = \begin{pmatrix} -\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}\sigma_1 \\ -\begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix}\sigma_3 \\ -\begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix}\sigma_1 \\ \begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix}\mathbf{i}_2 \end{pmatrix}. \quad \text{(6-a)}$$

We can define the "shape matrix", i.e., the matrix that will define the form of the space-time stream

$$m = \begin{pmatrix} x & O & O & O \\ O & xy & O & O \\ O & O & 2x & O \\ O & O & O & y \end{pmatrix},$$

where x and y are space coordinates. Applying this matrix on the transformed basis (6-a),

$$\begin{pmatrix} x & O & O & O \\ O & xy & O & O \\ O & O & 2x & O \\ O & O & O & y \end{pmatrix} \times \begin{pmatrix} -\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}\sigma_1 \\ -\begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix}\sigma_3 \\ -\begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix}\sigma_1 \\ \begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix}\mathbf{i}_2 \end{pmatrix} = \begin{pmatrix} -x\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix}\sigma_1 \\ -xy\begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix}\sigma_3 \\ -2x\begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix}\sigma_1 \\ y\begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix}\mathbf{i}_2 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} \\ \begin{vmatrix} 0 & ixy \\ ixy & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & j2x \\ j2x & 0 \end{vmatrix} \\ \begin{vmatrix} ky & 0 \\ 0 & -ky \end{vmatrix} \end{pmatrix}. \quad (7)$$

This is the same thing to

$$\begin{pmatrix} O & x & O & O \\ O & O & xy & O \\ O & O & O & 2x \\ y & O & O & O \end{pmatrix} \times \begin{pmatrix} \begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix} & O & O & O \\ O & \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & O & O \\ O & O & \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix} & O \\ O & O & O & \begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix} \end{pmatrix} \times \begin{pmatrix} +\mathbf{i}_2 \\ -\sigma_1 \\ -\sigma_3 \\ -\sigma_1 \end{pmatrix} =$$

$$\begin{pmatrix} O & x\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & O & O \\ O & O & xy\begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix} & O \\ O & O & O & 2x\begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix} \\ y\begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix} & O & O & O \end{pmatrix} \times \begin{pmatrix} +\mathbf{i}_2 \\ -\sigma_1 \\ -\sigma_3 \\ -\sigma_1 \end{pmatrix} = \begin{pmatrix} \begin{vmatrix} x & 0 \\ 0 & x \end{vmatrix} \\ \begin{vmatrix} 0 & ixy \\ ixy & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & j2x \\ j2x & 0 \end{vmatrix} \\ \begin{vmatrix} ky & 0 \\ 0 & -ky \end{vmatrix} \end{pmatrix}. \qquad (7\text{-a})$$

and for the sake of brevity I will presume the compact form (7-a) further on.

Each entry (*2×2* matrix) of the 4-D space-time field (7) represents a state of the system (still without consideration of induced flux[5]). We can assume that, under certain conditions, the states can form superpositions in different ways. Interestingly, one way to characterize such superpositions — including for later computations — is to introduce the 4-D Hadamard transform of the field, so that

[5] Obviously, if a spatiotemporal stream is induced, the state matrices will be set by the stream itself.

$$\left(\mathcal{M}\times\mathcal{T}\times\mathcal{B}\right)'=\frac{1}{\sqrt{4}}\begin{pmatrix}1_2 & 1_2 & 1_2 & 1_2\\ 1_2 & -1_2 & 1_2 & -1_2\\ 1_2 & 1_2 & -1_2 & -1_2\\ 1_2 & -1_2 & -1_2 & 1_2\end{pmatrix}\times\begin{pmatrix}\begin{vmatrix}x & 0\\ 0 & x\end{vmatrix} & \begin{vmatrix}0 & ixy\\ ixy & 0\end{vmatrix}\\ \begin{vmatrix}0 & j2x\\ j2x & 0\end{vmatrix} & \begin{vmatrix}ky & 0\\ 0 & -ky\end{vmatrix}\end{pmatrix}=\begin{pmatrix}\begin{vmatrix}\frac{x+ky}{2} & \frac{ixy+j2x}{2}\\ \frac{ixy+j2x}{2} & \frac{x-ky}{2}\end{vmatrix} & \begin{vmatrix}\frac{x-ky}{2} & \frac{-ixy+j2x}{2}\\ \frac{-ixy+j2x}{2} & \frac{x+ky}{2}\end{vmatrix}\\ \begin{vmatrix}\frac{x-ky}{2} & \frac{ixy-j2x}{2}\\ \frac{ixy-j2x}{2} & \frac{x+ky}{2}\end{vmatrix} & \begin{vmatrix}\frac{x+ky}{2} & \frac{-ixy-j2x}{2}\\ \frac{-ixy-j2x}{2} & \frac{x-ky}{2}\end{vmatrix}\end{pmatrix}. \quad (7\text{-b})$$

In this way, the field is represented on a different basis, with each component being a superposition of states, so that we can explore multiple states simultaneously.

On one hand, we note that in the supposed field (7) there are three matrix components with time signatures *i*, *j* and *k*. This means that we will have a 4-D space-time field with one pure spatial matrix-coordinate and three mixed matrix-coordinates with temporal signatures. On the other, from $\varphi(u,v)$ we have

$$\varphi_u=(1,0,-2u),\ \varphi_v=(0,1,-2v),$$

$$\varphi_u\times\varphi_v=\begin{vmatrix}e_1 & e_2 & e_3\\ 1 & 0 & -2u\\ 0 & 1 & -2v\end{vmatrix}=(2u,2v,1).$$

Now, from the above vector product we may construct, by the application of matrix (4), an ordered pseudo-Euclidian tetrad considering a pure temporal-signed quaternion[6], i.e., one to which the real part is zero (in reference to the zero thickness of the small region),

$$\begin{vmatrix} 2ui & 2v+i \\ -2v+i & -2ui \end{vmatrix},$$

thus proceeding a scalar product[7],

$$\begin{pmatrix} \begin{vmatrix} u & 0 \\ 0 & u \end{vmatrix} \\ \begin{vmatrix} 0 & iuv \\ iuv & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & j2u \\ j2u & 0 \end{vmatrix} \\ \begin{vmatrix} kv & 0 \\ 0 & -kv \end{vmatrix} \end{pmatrix} \bullet (0, 2ui, 2vj, k) \therefore$$

$$\mathcal{F}(u,v) = \iint_{\mathcal{S}} \left(\mathcal{T}_{4\times 4}(u,v) \times \mathcal{B}_4 \right) \bullet \mathcal{G}\left(\varphi_u, \varphi_v\right) dudv =$$

$$\iint_{\mathcal{S}} \left(\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} 2u^2 v + \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} 4uv + \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} v \right) dudv =$$

$$\int_0^1 \int_0^1 \left(\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} 2u^2 v + \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} 4uv + \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} v \right) dudv =$$

$$\int_0^1 \int_0^1 \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} 2u^2 v dudv + \int_0^1 \int_0^1 \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} 4uv dudv + \int_0^1 \int_0^1 \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} v \, dudv =$$

[6] Note that we are treating with a small surface portion of a hypothetical metamaterial, and this transformation comes from the action of an external agent such as an electromagnetic field.

[7] By the scalar product, the temporal signature becomes a unitary real value, allowing the determination of a real quantity associated with the time-like stream.

$$\begin{vmatrix} 0 & -2 \\ -2 & 0 \end{vmatrix} \int_0^1 \frac{v dv}{3} + \begin{vmatrix} 0 & -4 \\ -4 & 0 \end{vmatrix} \int_0^1 \frac{v}{2} dv + \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} \int_0^1 v dv = \begin{vmatrix} 0 & -1/3 \\ -1/3 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} + \begin{vmatrix} -1/2 & 0 \\ 0 & 1/2 \end{vmatrix} = \begin{vmatrix} -1/2 & -4/3 \\ -4/3 & 1/2 \end{vmatrix}. \quad (8)$$

The matrices inside the double integration indicate the interaction state of the stream with the metasurface, while the constants that multiply them establish the intensities of the couplings with the structure of the metamaterial. The final matrix (8), named the "flux matrix" of the quaternion field $\mathcal{T}_{4\times4}(u,v)\times\ \mathcal{B}_4$, has modulus of its determinant given by

$$\left\| \det \left| \mathcal{F}(u,v) = \iint_{\mathcal{S}} \left(\mathcal{T}_{4\times4}(u,v)\times\ \mathcal{B}_4 \right) \bullet \mathcal{G}(\varphi_u, \varphi_v)\, du dv \right| \right\| =$$

$$\left\| \det \begin{vmatrix} -1/2 & -4/3 \\ -4/3 & 1/2 \end{vmatrix} \right\| = 2.03\,. \quad (9)$$

This result is interpreted as the "stream" of space-time through a hypothetical small metamaterial singular region — a metasurface — without a pure spatial component. In ordinary four-dimensional space, all directions coexist in linear time. In the tetrad of three times and one spatial dimension, past, present, and future coexist in one-dimensional space. Each small interval of the space-time flow line carries the conjunction of its past, present, and future. This representation can be useful if associated with the generation of quantum entanglement by the metasurface and the manipulation of the entangled quantum states along the space-time stream through the metasurface[8].

Let us take another case in which we consider the "bridge" embedded into the fourth dimension (possibly the very home of gravity),

[8] The electromagnetic response of the metasurface can be controlled, for instance, by a geometric-phase mechanism based on the Pancharatnam-Berry phase, having the metasurface as a platform for quantum information.

$$\begin{pmatrix} O & x\begin{vmatrix} 0 & -i \\ -i & 0 \end{vmatrix} & O & O \\ O & O & xy\begin{vmatrix} +j & 0 \\ 0 & +j \end{vmatrix} & O \\ O & O & O & 2x\begin{vmatrix} +k & 0 \\ 0 & +k \end{vmatrix} \\ y\begin{vmatrix} 0 & -\varepsilon \\ \varepsilon & 0 \end{vmatrix} & O & O & O \end{pmatrix} \times \begin{pmatrix} (1_2, \mathrm{E}_1) \\ +i\sigma_1 \\ +j\sigma_1 \\ +k\sigma_3 \end{pmatrix} = \begin{pmatrix} +x1_2 \\ -xy\sigma_1 \\ -2x\sigma_3 \\ y(\sigma_2, \mathrm{E}_3) \end{pmatrix}. \quad (10)$$

The parameterization of x and y yields

$$\begin{pmatrix} \begin{vmatrix} u & 0 \\ 0 & u \end{vmatrix} \\ \begin{vmatrix} 0 & -uv \\ -uv & 0 \end{vmatrix} \\ \begin{vmatrix} -2u & 0 \\ 0 & 2u \end{vmatrix} \\ \begin{vmatrix} 0 & -v(i+\varepsilon) \\ v(i+\varepsilon) & 0 \end{vmatrix} \end{pmatrix} \bullet (0, 2ui, 2vj, k) \therefore$$

$$\mathcal{F}(u,v) = \iint_{\mathcal{S}} \left(\mathcal{T}_{4\times4}(u,v) \times \mathcal{B}_4 \right) \bullet \mathcal{G}(\varphi_u, \varphi_v)\, dudv =$$

$$\iint_{\mathcal{S}} \left(\begin{vmatrix} 0 & -i \\ -i & 0 \end{vmatrix} 2u^2v + \begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix} 4uv + \begin{vmatrix} 0 & -k \\ k & 0 \end{vmatrix} v(i+\varepsilon) \right) dudv =$$

$$\int_0^1 \int_0^1 \left(\begin{vmatrix} 0 & -i \\ -i & 0 \end{vmatrix} 2u^2v + \begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix} 4uv + \begin{vmatrix} 0 & -k \\ k & 0 \end{vmatrix} v(i+\varepsilon) \right) dudv =$$

$$\int_0^1 \int_0^1 \begin{vmatrix} 0 & -i \\ -i & 0 \end{vmatrix} 2u^2v\, dudv + \int_0^1 \int_0^1 \begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix} 4uv\, dudv + \int_0^1 \int_0^1 \begin{vmatrix} 0 & j \\ -j & 0 \end{vmatrix} v\, dudv =$$

$$\begin{vmatrix} 0 & -2i \\ -2i & 0 \end{vmatrix} \int_0^1 \frac{v\,dv}{3} + \begin{vmatrix} -4j & 0 \\ 0 & 4j \end{vmatrix} \int_0^1 \frac{v}{2} dv + \begin{vmatrix} 0 & j \\ -j & 0 \end{vmatrix} \int_0^1 v\, dv =$$

$$\begin{vmatrix} 0 & -i/3 \\ -i/3 & 0 \end{vmatrix} + \begin{vmatrix} -j & 0 \\ 0 & j \end{vmatrix} + \begin{vmatrix} 0 & j/2 \\ -j/2 & 0 \end{vmatrix} = \begin{vmatrix} -j & -i/3 + j/2 \\ -i/3 - j/2 & j \end{vmatrix}. \quad (11)$$

From the flux matrix (11), we get

$$\left\| \det \left| \mathcal{F}(u,v) = \iint_{S} \left(\mathcal{T}_{4\times4}(u,v) \times \ \mathcal{B}_4 \right) \bullet \mathcal{G}\left(\varphi_u, \varphi_v\right) du dv \right| \right\| =$$

$$\left\| \det \begin{vmatrix} -j & -i/3 + j/2 \\ -i/3 - j/2 & j \end{vmatrix} \right\| =$$

$$\left\| 1 - \left\{ \left[-i/3 + j/2 \right] \bullet \left[-i/3 - j/2 \right] \right\} \right\| =$$

$$\left\| 1 - \left\{ -1/9 + k/6 + k/6 + 1/4 \right\} \right\| =$$

$$\left\| 1 - \left\{ 5/36 + k/3 \right\} \right\| =$$

$$\left\| 0.86 + 0.33k \right\| =$$

$$\sqrt{0.74 + 0.11} = 0.9219 \ . \qquad (12)$$

Evidently, due to the thought habit guided by classical mechanics, the difficulty in imagining a flow of space-time is because at no point in my description we must think of an abstract theoretical fluid as in the models we are familiar with. Physicists soon think of energy density, incompressibility, etc., characterizations that would not apply to a single, primordial entity, prior to all ponderable material existence, "flowing" *per se* and everywhere. Only space-time has inherent existence, that is, it does not consist of parts; the perfect monad. Those properties later appears as emergent features of the complex interactions between expanding space-time neighborhoods conflicting with one another. In fact, there is no material fluid, not even in the classical sense. There is only space-time in stream motion. In this way, the insight I had was that gravity is a feature of the flow of space-time itself. The first stage immediately after the pure space-time existence includes mass as an emergent property, which means that gravity is in principle not dependent on mass. Although we do not know the deeper details of those interactions and the emergence, it is worth theorizing on the possible implications of these ideas for the Standard Model and the understanding of dark energy.

That's what space-time is like. At every tiny interval, there is an expansion that is unequal to the others in its surroundings. Those tiny expanding cells conflict with one another in a random manner. From such conflicting interactions, circumstances and properties arise that would not otherwise exist. Expansion is primordial; mass is emergent.

2.1 A semiological enhancement

Previously, I defined the operation "sharp-product" (#), which is nothing more than a product identified as modularization between two quantities, constituting a term considered fundamental to the physical meaning of the formalism in question. Therefore, it is worth rewriting the flow integral by applying the #-operator adapted to the matrix and scalar products, understanding their respective roles:

$$\mathcal{F}(u,v)=\iint_{\mathcal{S}}\left(\mathcal{T}_{4\times4}(u,v)\overset{\otimes}{\#}\ \mathcal{B}_4\right)\overset{\bullet}{\#}\mathcal{G}(\varphi_u,\varphi_v)\,dudv\,.$$

Briefly, while the sharp-matrix-product, $\overset{\otimes}{\#}$, defines "what" flows, the sharp-dot-product, $\overset{\bullet}{\#}$, establishes how to stream.

Continuing the discussion, needless to say that the flow of space-time is independent of cosmology, being what space-time does by its own nature. Thinking of an arbitrary sub-Planckian geodesic interval as a small reservoir of energy, not all of this energy translates into expansion. According to what I said in the previous sections, part of that energy is hidden in the "fifth dimension" embedded into the fourth — the "bridge" — in the form of a “dark” gravitational energy, preventing the continuum from being torn apart by expansion and being released in the presence of mass (it would not be unreasonable to suppose that the “fifth dimension”, "which is neither space nor time," is the very realization of gravity).

In conclusion, the approach I have taken is more fundamental than the search for solutions to Einstein's equations in the sense that it is not the general cosmic structure that is at stake, but rather the integrative nature of this structure. It may represent a relevant initial step towards more viable future technical developments in terms of spacecraft displacements.

For a long time now, the proliferation of the "literature of the obvious" has been observed, a complete lack of imagination that only fuels indifference and mediocrity. It is often read, "Scientific activity is a collaborative process", as if were a surprising observation! Who could object to such an obvious truth? Collaboration is the foundation of human evolution. However, there is a certain generality embedded in the

understanding of what “collaboration” means. Although there is no interpersonal dialogue, a solitary scientific activity taking existing knowledge forward and committed to publishing its results, seems to me to be quite collaborative in the sense that several authors have recorded their knowledge in the literature accessed, studied and honestly cited. I think that the exaltation of *stricto sensu* collaboration in recent times raises some keynote points to pay attention: 1) I have noticed the growth of a kind of hive thinking, largely frustrating and harmful to any attempt at disruptive renewal in science as it is today. This type of collaborative work reflects a negative ideological model, and closes the doors to innovation, discouraging creative minds from approaching the academic environment; 2) scientific corporatism in defense of economic interests and related to the crystallization of conditions for the exercise of power also grows as a side effect of the neoliberal market and governance models; 3) in either of the two previous situations, the effect of fear of failure is apparently mitigated when responsibility falls on a hive and not just on an individual[9]; 4) it cannot be ruled out that the academic standardization of thought serves the interests of the large corporations that dominate the world and the governments that court them; thinking critically is not a welcome practice in the world of consumption.

Sometimes I wonder if the discouragement of solo initiatives is accompanied by intentions other than those of true science in its original state and in its guiding principles. Hive thinking originates from a primitive state called "social induction," that is, "if everyone is doing the same thing, then it must be the right thing to do." Thus, anyone who takes an alternative path is automatically excluded from the hive. Finally, hive thinking ends up constituting a nucleus of authoritarianism, a true center of exclusion. I can imagine how many good individual initiatives have been lost due to personal vanity and academic hypocrisy.

I faced some ridiculous situations of moral outrage before I established myself as a scientist because I was always less sociable than most. This feature of my personality does not mean that I am an entire solipsist (although I have my moments of almost monastic isolation). On the contrary, I work with some great partners who share my convictions. However, we do not take part in closed science clubs, nor do we have the slightest interest in such participation. Science must be open, dynamic, creative, inviting and close to society. Strictly speaking, it was never exactly like that, but at least if we compare scientific activity today with that of the first half of the 20th century, we easily reach the conclusion that, from the point of view of man's inventive and innovative capacity, science is relatively stagnant. Given the many factors responsible for this indisputable truth, I will mention just three I consider the most relevant: 1) the systematic academic rejection of individual creative initiatives, 2) the pressures of consumerism, homogenizing culture at a very low mental

[9] In science, there is no point in being afraid of failing, unless someone sees it as a path to foolish success. Showing that something does not work is also a positive result for the advancement of ideas.

level, and 3) the excessive politicization of academies and education in general. In particular, with reference to the second, there is an insidious confusion between technological advances (especially for easy and widespread consumption) and scientific progress[10], notably in physics. Following in agreement with Mario Bunge [6], science does not emanate technique. The technician uses basic science, but science does not design useful artifacts, nor does it talk about costs. Thus, the technician has to deal with the problems that arise to him, which are rarely of interest to science. Furthermore, if there is one thing that physics teaches us it is that we have limits imposed by laws of nature that cannot be circumvented, no matter how greedy we are for technological improvements, and regardless of future general advancements that may occur. Due to the mistaken view that technological advancement depends on progresses in physics (or science), it is expected that, in the context of people's daily lives, technologies replace science to some extent because of its undeniable efficiency in deceiving people with promises of an easier life with low intellectual cost, a terrifying prediction, but very plausible.

2.2 The reentrant galaxies

Sci-fi cinematographic creations can be very suggestive, as I have noted. In the third season of Star Trek: Picard, an unlikely and incredibly conceived scene from the first episode shows the iconic Captain Jean-Luc Picard, then a retired Admiral, writing at his desk with a fountain pen. Just think, at the beginning of the 25th century, someone, in a truly nostalgic environment, using a writing instrument designed centuries ago! Whoever the creators of that splendid sequence of images were, I believe the message conveyed is that we should never lose the best of our humanity.

Searching the internet, I found a beautiful unbranded trio (fountain pen, mechanical pencil and pocketknife, all with a pretty good *guilloché* finishing) in thick layers of 14K gold, probably from the 1920s (Figures (6)). The fountain pen is excellent, few signs of use, 14K gold fine-to-medium nib and piston-filler with a great glassy ink reservoir. I bought the set at an affordable price, well in line with my discussion of the price/value relationship.

[10] Some claptraps arise from this confusion, such as the intellectual comparison between Silicon Valley and the Copenhagen School.

Figures (6) - The nice unbranded gold-filled trio.

The ink flows perfectly, even if the pen is left unused for several days. I decided to use it to start writing this section, as well as to illustrate the model I am going to discuss (by the way, fountain pens are great for illustrating, mainly fine and fine-to-medium nibs!).

The "reentrant galaxies" model, a topological approach, comes in response to the so-called “impossibly early galaxy problem”, a serious puzzle after recent confirmation of many massive galaxies in twilight of their lives observed at $z \sim 3$, when the universe was only ~ 2 Gyr old. This disconcerting fact calls into question the consensually accepted theory expressed by the hierarchical ΛCDM model, in which galaxy assembling, a long term gradual merging of several progenitors, took place at $z \sim 0.7$, when the universe was ~7 Gyr old.

Much of our understanding of the universe grounds on spherical or near-spherical symmetry, of course, because of the way gravity acts. Although this is a natural basic assumption, gravity itself can introduce, under certain extraordinary conditions, strange curvatures to that symmetry without losing it completely. Supposedly, the cosmic woof can expand within a time-bulb, a kind of cosmic conduit along which from a given moment the four-dimensional sphere begins to fold inwards on itself[11], creating a depression in space-time woof such as pressing a rubber balloon with a finger (a sphere with a *hilo*, or, as I called it, a *hilosphere*). Recalling Poincaré's conjecture, as there are no topological holes, any closed arc on the *hilosphere* surface can be continuously reduced to a point. Thus, we may consider it as homeomorphic to a four-dimensional sphere. The hypothesis is that the first oldest galaxies would be dragged by the *hilo* — the reentrant warp — "toward the past", diving into the depression asymptotically until a horizon that inexorably separates us from the ground zero. Note that the theoretical time-bulb expands in all directions, including the reentrant direction — the *hilo* —, which leads to complex and exotic compositions between redshifts[12]. It seems to me that it is a model capable of shedding light on several issues, but at the cost of some review in the essence of the standard model (not necessarily replacing it). Although we do not know why the *hilo* is formed, we can think of a kind of delayed extreme gravitational effect caused by the Big

[11] The conduit represents the historical sequence of expansion states of the quadrisphere.

[12] Between a normal and a reentrant galaxy, there is the redshift referring to the deviation due to the global expansion of the bulb, and the redshift referring to the deviation due to the reentrant expansion, i. e., the inner expansion of the *hilo*.

Bang itself. The fact is that new records of old galaxies will probably populate our compiled surveys, perhaps not many. In any case, the hierarchical model of galaxy formation is currently in check. The reentrant galaxies' model shows how we can build radically different solutions without losing rationality and contact with reality through observation. Certainly, it is not a proposal that aligns with the hive thinking that has taken hold in academia.

The schematic drawing of the reentrant galaxies model shown in Figure 7 was recorded in my handwritten notebook. This is the second version of the model representation, which I made a point of recording among equations and theoretical discussions for posterity.7

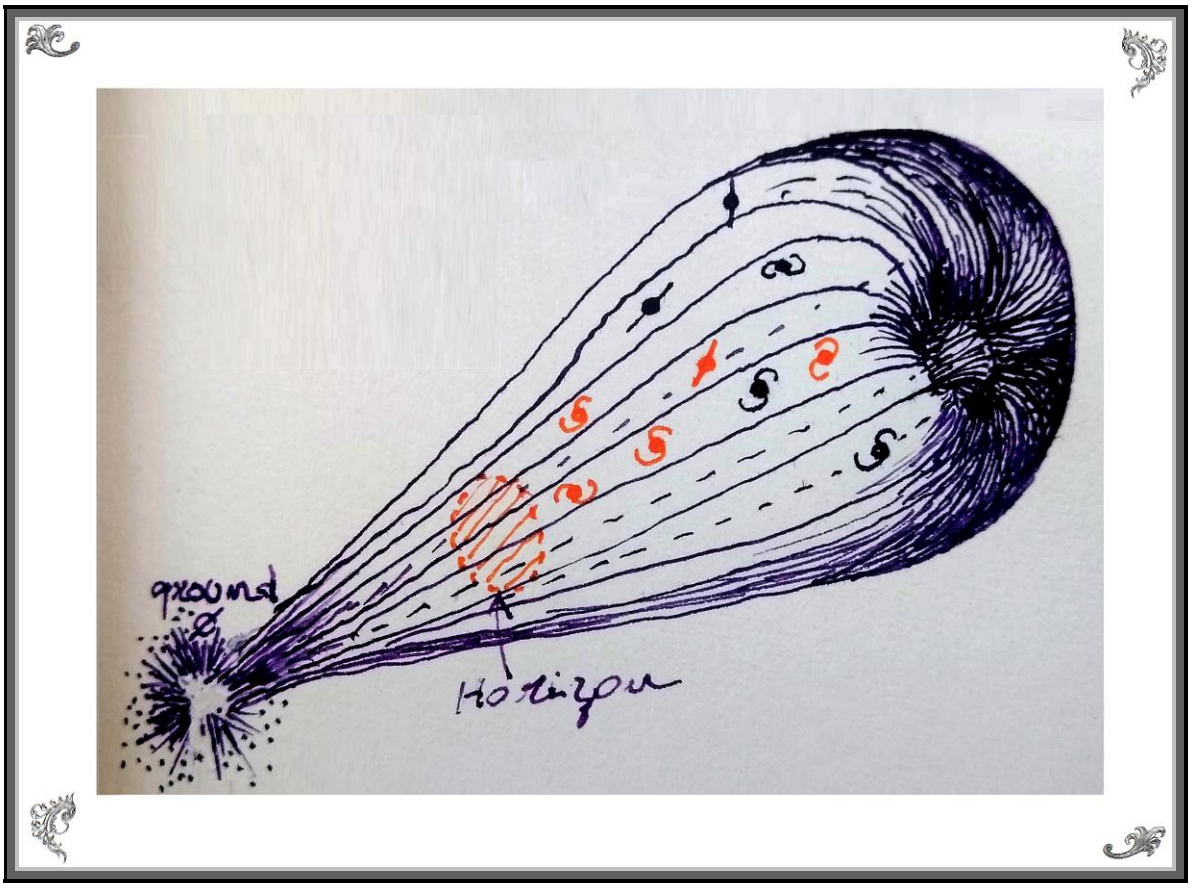

Figure 7 - The schematic model of reentrant galaxies in its first approach pictured by the unbranded piston-filler, 14K gold-filled, fountain pen from Figures (5). Note that the time-bulb expands into the future with the galaxies in black emerging from a time after the horizon of approach of the galaxies in orange, towards the past. For this reason, we see mature galaxies where they could not be if we considered only the most intuitive evolutionary part of the universe.

A welcome interference! In this section, I am including two recent acquisitions at very comfortable and, in my opinion, fair prices: Two deluxe vacumatics (Figures (8), Figure 9 and Figure 10). The nomenclature of vacumatics is extremely confusing. I am never sure of the models. Fortunately, thanks to Nishimura [10], I was able to identify my items more accurately. The "deluxe" ones, top of the line, remained until the end of the 1930s, being of the double-jewel type, with two-tone nibs (platinum plated nibs topped by the famed golden arrow inherited from the original 1932 Golden Arrow). The two I bought are entirely made in excellent green-pearl laminated celluloid — including the jewels, striped in the same manner —, one of them slender, fine-to-medium nib, aluminum normal plunger, with the new arrow clip and the obscure and coveted star on the top of the clip, only applied during a few months in 1939 (apparently, the application technique was unfeasible for large-scale production). The star would most likely have been the first symbolic proposal of quality warrant, replaced shortly afterwards by the iconic blue diamond. The other, standard, normal "archers arrow" clip introduced in 1932, equipped with an aluminum locking plunger and the same two-tone fine-to-medium nib. Both pens in perfect condition. The rarity of these models in so good condition, especially the first, makes them expensive items if they are found on eBay or other commercial sites. From the 1940s onwards, except for some isolated transition model, vacumatics were restricted to formats with rounded blinds and monochromatic nibs with no golden arrow until they phased out in United States at 1948 (in Canada production continued until 1953). In this era, clips may or may not feature the blue diamond.

Figures (8) – From top to bottom: Deluxe slender (1939) and Deluxe standard with "archers arrow" clip, both with bicolor nibs.

Figure 9 – The rare star imprint on the clip of 1939 slender.

Figure 10 – The "archers' arrow" clip of the standard.

3 Searching murmurs from outside

I have already written many pages on the issue of extraterrestrial life, especially intelligent life, before reading the beautiful book of my colleague Ćirković [7], a truly inspiring work. I will give here a quick summary of my opinions, showing in a realistic way our condition of existence limited to the only world that we know to be a perfect shelter for life.

It seems clear that not all types of intelligence produce culture. We have examples right here on Earth. Dolphins and whales are highly intelligent, as are octopuses. Culture is an artificial — that is, non-biological — and extremely sophisticated form of adaptation based on the creation of implements. Man adapts to the world and its changes through culture. On the other hand, we can reasonably assume that intelligences adapted through culture must have some similarities, however strange their biological forms may be. For example, competition for resources in use of complex tools and implements seems to be something spontaneous in cultures that are successful in occupying the available geographic spaces. However, if they are successful, they will occupy increasingly larger portions of living space, consuming resources at growing rates. Sooner or later, planets will exhibit environmental stress from continued exploitation and concomitant waste generation. There will be conflicts and disputes over territories, without even speculating about the nature of the alien psyche in terms of desires for power, ethnic prejudices, etc. Thinking only from the perspective of survival, with ideals of hegemony prevailing over living space and resources, there may not be enough time for a civilization to develop sufficient technology to take its hegemonic intentions beyond its home planet. It will disappear before it can be noticed. This, in fact, seems to be the way we are on. We may even last a few more centuries, or even a few millennia, but I do not believe we will get very far.

As attractive as the belief in the existence of other civilizations in the Cosmos is, the laws of nature seem to conspire against so that we either never confirm the expectation (because other civilizations simply do not exist) or, if confirmed from a distance, we never come face to face with aliens (in addition, I believe that only in a narrow range of expansion of the universe is life possible, which further reduces the chances of the emergence of intelligence). Based on what we know from history about the encounter between people with very different technological levels, stellar civilizations have a better chance of survival by remaining isolated on their homeworlds. I would not bet on benevolence and kindness from exointelligences. Fortunately, the natural obstacles to an encounter are such that we can consider ourselves saved to a certain extent (as long as we do not create a motivation for someone!).

All the technological, biological and physical problems we have identified are universal constraints to be faced by any intelligence; cosmic radiation, efficient technology to leave planet's surface, adaptation of organisms to the

dangers of space, radical interstellar transport technology, food sustainability, advanced navigation resources (we are used to seeing spaceships flying across the outer space in the movies as if they were airplanes. Let there be physics! Imagine a sharp change of route in a ship traveling at sub-light speed!), and many others. For us, all these problems are open. Perhaps, for a civilization ten or twenty thousand centuries ahead — which is a blink of an eye in the evolution of the universe —, such problems are not even understood as real problems, but I do not believe it. As I have already pointed out, technology is finite and conditioned by natural laws. There is also a myth, contributing to the general ignorance regarding this unquestionable fact, coming from classical economics, that "man will always be able to find new sources of energy and to invent new ways of dominating them, for his benefit", a belief inspired by a mechanistic model of the world, from which everything is reversible. This mistaken and fanciful model, which should actually give way to one inspired by thermodynamics, has brought us the current crisis, and is still around among many.

The romanticism of the Voyager mission would presumably put us at great risk if it were not for the immensity and hostility of the cosmic void, and the more than likely non-existence of intelligent beings as far as our instruments reach across our galaxy. The vacuum is so vast that tens and hundreds of thousands of years separate the sun from the nearest stars beyond *Proxima Centauri*. Even so, I still think one of the smartest things about the Voyager mission was not including a lovely, primitive and genuinely human utensil called "fountain pen"; an advanced civilization that could understand it as a communication instrument could feel greatly encouraged to carry out an *a priori* successful conquest operation.

Looking at the pen case closest to where I am sitting, I noticed an old Wearever that I have hardly ever used (Figures (11)). Wearever was a brand frequently associated with cheap and shoddy pens, the kind you'd buy only in an emergency to jot down a few quick notes on the way to the airport (detractors of this brand like to repeat tirelessly that they are cheap pens). Although some models of David Kahn's pen manufacturing were indeed of poor quality, my experience has shown that a causal relationship between price and quality cannot always be established. The cheap item I have, a Deluxe probably from the mid-1930s, considered top of the line, was made of excellent material and has survived the test of time in perfect condition, including the gold plating of the metal parts (it clearly tries to follow the models of famous brands). I think I will use it a bit in the last section.

Figures (11) – The not so liked Wearever.

4 On the sense of usefulness

If someone asked me what Einstein's general relativity is for, I would say that, regardless of any future usefulness, it serves to help us understand eternity (in a beautiful scientific way), and to learn to contemplate it. In this way, we can realize the finiteness of life and of known things, as well as our fleeting role in the eternal scenario. Nevertheless, the sense of usefulness varies from person to person. Very few are interested in contemplating eternity, which is lamentable.

Regarding the usefulness of old fountain pens, I will try not to repeat myself. Just as much as the intellectual work of reading, reasoning and mental elaboration of ideas, the materialization of literature on paper is an artisanal activity. Even more artisanal is the use of writing instruments for which the supply and maintenance are very craft-like. I think that choosing such instruments is a question of aesthetics, important for some, indifferent for others. In any case, the intellectual who knows the price of building a model or theory also knows the value of patience and playful means of relieving mental tensions, things that can be exercised daily with a modest collection of old fountain pens.

5 Final words

Over the years, writing with old fountain pens has been a pleasurable and, at the same time, invigorating habit for disruptive thinking. I have a transcendental relationship with these charming relics. Certainly, they are and always will be an essential part of my motivation as a theoretical physicist and philosopher. Some, more fragile, require constant attention; others, wait to be asked as if they had never slid onto the paper. It is fun to take care of them all during the intervals of mind's rest. I do not think it would be the same to do science any other way.

With respect to old fountain pens, I dislike the commercial view that everything was created in Western Europe and North America. This is not true. As my Romanian colleague and friend Viorel Badescu noted, the first patent on record belonged to the Romanian Petrache Poenaru, a mathematician, physicist, engineer and inventor, among other skills, who, meeting his own extreme writing needs, invented the fountain pen and registered it on 25 May 1827, at the Manufacture Department of the French Ministry of the Interior under the code number 3208 and described as "*plume portable sans fin, qui s'alimente elle-meme avec de l'ancre*" (Figure (12)). Another example, Pelikan acquired the patent of the Croatian chemist Slavoljub Eduard

Penkala, and the patent of the Hungarian engineer Theodor Kovacs' piston filler system. Then, it seems that the Balkans[13] are more present in fountain pen history than one would imagine. Furthermore, many famous brands made use of technologies developed by forgotten men and little-known brands.

Figures (12) – The patent registration of the Romanian inventor Petrache Poenaru (credits to INPI - *Institut National de la Propriété Industrielle*, available at: http://bases-brevets19e.inpi.fr/).

[13] Although there is no consensus, I prefer to include Romania and Hungary in the Balkans for historical-cultural reasons.

As for the direction of science, I do not like what I see. There is too much concern with knowledge and too little with understanding. This contributes greatly to the current relative stagnation of scientific activity, and it seems that no one notices or cares about it. I am a defender of the idea that our universe is essentially simple and predictable on both the largest and smallest scales, as respectively our most powerful telescopes and the Large Hadron Collider, LHC, have repeatedly shown, the latter in particular without having pointed out any deviation from the known physics on the smallest scales. I have never underestimated the "Occam factor", which logically suggests that we should discard models and theories with many parameters and possibilities, giving way to the simplest and most mathematically elegant systems of hypotheses. In fact, there is nothing that forces us to accept complicated and poorly predictive models, tying us down by mere authoritarianism to ideas that have not led to the clarifications we have been seeking for decades. During this time, several experimental and observational attempts have not revealed any evidence that could justify string theory (although I consider it an impressive construction). The results have always pointed to a simpler universe. In this way, our brief time of existence, I believe, will be better spent on alternatives to the standard orthodoxy, which promise results that are more rewarding.

Yet, what is coming if we insist to ignore the flops? I do not know. The effort should be to focus on understanding, not on the accumulation of knowledge or on the scientific sectarianism, which creates a true straitjacket. There is an excess of information provided by the Internet making the world scattered and people unable to focus, and, if we consider the heavy publicity for generative AI (a form of authorized plagiarism), things seem to be heading rapidly towards a deteriorating outcome.

It is a mistake to think that science is immune to the social follies of humanity. The mind is erratic and susceptible to subliminal stimuli and ego addictions. No one is immune. In today's world, civilization writes the chapters of its time on Earth by computational typing within a materialistic-hedonistic agenda in which practical things of immediate use, far beyond real needs, stand out in flashing signs. This has immediate repercussions, whether conscious or not, on all our choices. Seeking illusions in poorly elaborated thoughts, it is easier to repeat the naive adage that the universe is mathematical rather than to face it in all its constraining physical truth. Thus, cosmology taken to the extreme of seriousness does not seem appealing outside of a very restricted intellectual environment. However, if we live long enough, we shall be able to enjoy some direct and indirect benefits of this science that has only gained strength and respect in the last few decades. It certainly helps us understand the relevance of the two special days in our lives as Twain pointed out: the day we were born and the day we discovered why.

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